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Price War Analysis Using a Variant of the Richardson Model

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Summary: *We present a model for movement of prices of a product of two competing firms during a price war. The Richardson's model for arms race is modified to yield a system of differential equations. The equations are analyzed using numerical techniques along with nonlinear theory. It is observed that prices in a price war tend to stabilize after some point. Managerial implications of this study are discussed.*

Key words: *Price war, mathematical modeling, differential equations.*

Notation:

$x = x(t)$ is the price of firm A's product at time t.
$y = y(t)$ is the price of firm B's product at time t.
x_{max} is the maximum price of firm A's product.
y_{max} is the maximum price of firm B's product.

1. INTRODUCTION

An essential condition for a price war is inter-firm rivalry. Cassady [4] sees a price war as an engagement between two or more firms who use price as a weapon by making successive moves and countermoves to gain advantage in the market or even resist an advantage gained by the other firm.Telser [9] considers price war as a strategy of reducing the prices by the bigger or financially stronger firm to push the smaller firm outside the market. Heil and Helsen [6] bring out a list of conditions required for a price war. These are:

a) The main focus of the pricing actions and reactions is on the competitor and never on the consumer.
b) This interaction is typically undesirable to the competitors.
c) The competitors never thought of starting the price war through their behavior.
d) The pricing interaction occurs at a faster rate than normal changes in prices.
e) The direction of pricing is always ‘downward’.
f) This pricing behavior is completely unsustainable.

In this paper, we have analyzed the price movement of two firms engaging in a price war. A variant of the Richardson’s model for arms race [8] has been used to model the price movement of the competing firms. Our model consists of a pair of simultaneous nonlinear differential equations for which numerical solutions have been obtained using RungeKutta method of order 4. The model can also be generalized to include the case of k firms indulging in the price war.

2. MODEL FOR TWO FIRMS

Our objective in this section is to obtain a model for the price movements of a fixed product for two firms A and B indulging in a price war. We base our model on the following assumptions:

(i) The rate of change for the price of firm A is correlated with the price of firm B. At the initial stage the realization between the firms regarding existence of the price war will not be present. At this point the rate of change of A will be slower. When the price war will start raging and prices have become appreciably lower the rate of change of A will be faster. Similar logic applies for the rate of change for B.
(ii) The rate of change for the price of firm A and B is directly proportional to their current price.
(iii) There is an independent factor based on mutual collusion or suspicion between the two firms which affects their rate of change of prices.
(iv) There is an upper limit beyond which the prices of A and B cannot increase. Exceeding this limit would mean termination of the price war.

Using the above assumptions we propose the following model to consist of the following pair of differential equations:

$$\frac{dx}{dt} = \left(1 - \frac{x}{x_{max}}\right)\left(-\frac{a}{y} + mx - r\right)$$

$$\frac{dy}{dt} = \left(1 - \frac{y}{y_{max}}\right)\left(-\frac{b}{x} + ny - s\right)$$

Here,
a is the reaction factor by which firm A reacts to firm B's changing of price.
b is the reaction factor by which firm B reacts to firm A's changing of price.
m is the fatigue factor for firm A with respect to the price war. If $m > 0$ firm A desires to end the price war. If $m = 0$ firm A is non-committal towards the price war. If $m < 0$ firm A desires to perpetuate the price war.
n is the corresponding fatigue factor for firm B.
r is the grievance factor for firm A. If $r > 0$ then firm A is highly aggrieved towards firm B.

Similarly $r = 0$ and $r < 0$ indicate neutrality and goodwill of firm A towards firm B respectively. s is the corresponding grievance factor for firm B.

Some remarks are in order. Consider the first differential equation of our model:

$$\frac{dx}{dt} = \left(1 - \frac{x}{x_{max}}\right)\left(-\frac{a}{y} + mx - r\right)$$

The terms on the right hand side may be understood in terms of our stated assumptions. In particular the term $\left(1 - \frac{x}{x_{max}}\right)$ imposes an upper limit for the price as per assumption (iv) and is justified by the standard technique of converting an exponential growth model into a logistic growth model. This model derives its intuition from Richardson's original model for arms race [8] and its modification by introducing the carrying capacity term in [7].

2.1 Solution of the Model

The chief difficulty of our model is that it consists of a pair of simultaneous nonlinear differential equations which are not easily solvable. For practical purposes it suffices to use a numerical method to obtain a solution. For the purpose of illustration we have considered two situations and used Runge-Kutta method of order 4 to obtain approximate numerical solutions [1].

In the first situation we have assumed a case when both parties are willing participants in a price war (i.e. $m, n < 0$). We have let:

$(x_{max}, y_{max}, a, m, r, b, n, s, x_0, y_0) = (20,20,0.3,-0.5,0.1,0.8,-0.6,1,4,10)$ where x_0, y_0 are the initial prices for firm A and B respectively.

Application of the Runge-Kutta method with step size $h = 0.1$ now yields values of $x(t)$and $y(t)$. We present this on Figure 1 and Figure 2 by the following price curves for $x(t), y(t)$:

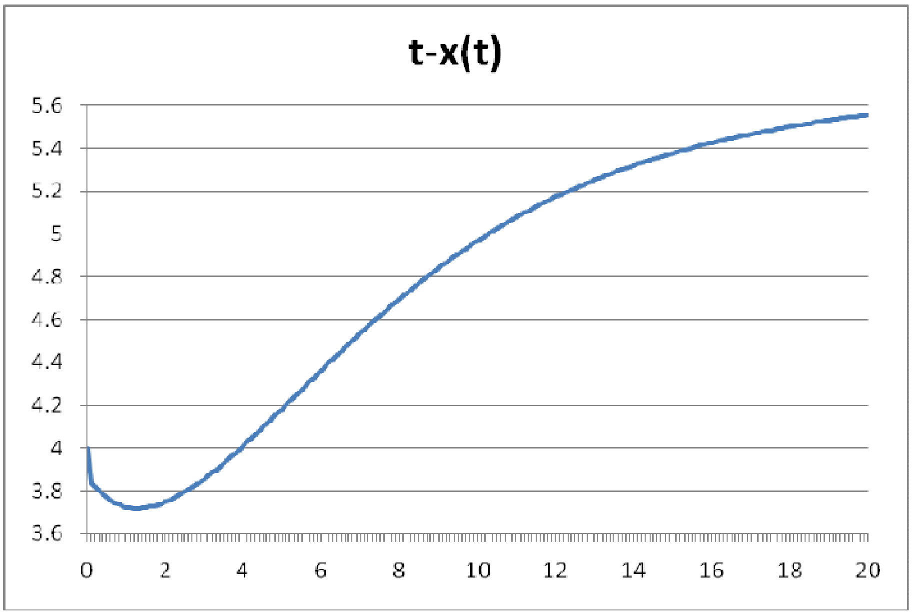

Figure 1. Price curve for x(t)

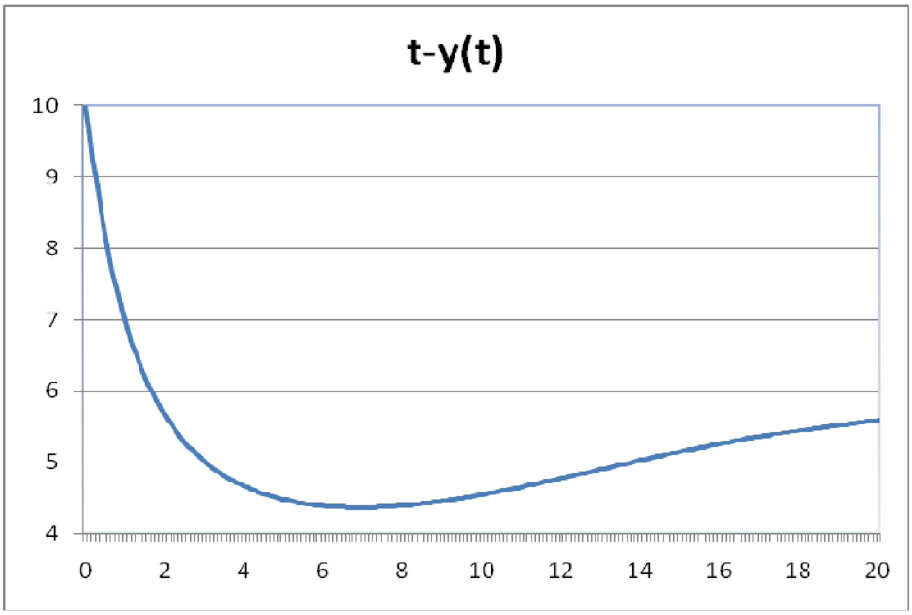

Figure 2. Price curve for y(t)

As is visible from the graphs above, prices plummet to a certain bottom line at the initial stages of a price war when there is great enthusiasmand emotions running between the firms which are caused by the violation of accepted competitive behavior. Prices of firm B plummet for a longer time as initially firm B had priced its product higher so it takes some more time to adjust itself near its bottomline. To sustain themselves both firms then are forced to slightly increase their prices and gradually settle themselves near a stable equilibrium of around 6 for their prices. Again it is worthwhile to note that firm A is increasing its prices faster than B. This is on account of A being more quickly fatigued of the price war: its fatigue factor is greater than that of B. Again A is less hostile towards B than B is towards A as $r < s$. This translates to A being less competitive in the price war and more desirous of prices being pushed up for normalcy to prevail.

We may further analyze our model by using nonlinear theory in the following fashion: Our computations have suggested (as do the above graphs) that $(5.7,5.9)$ is an equilibrium point at which $\frac{dx}{dt}, \frac{dy}{dt}$ vanish. We may now linearise our system using standard nonlinear theory [2] by considering the Jacobian matrix for our system and evaluating it at $(5.7,5.9)$. This yields the matrix:

$$\begin{pmatrix} -0.648588 & 4.87515 \\ 0.00102596 & 4.87515 \end{pmatrix}$$

Since the determinant of the above matrix is negative so we conclude that the equilibrium point thus obtained is a saddle point.

In the second situation we have taken the case when only one party is willing to participate in the price war while the other party is unwilling (i.e. $m > 0, n < 0$).
We have let $(x_{max}, y_{max}, a, m, r, b, n, s, x_0, y_0) = (6,6,3,1,1,2,-5,0.1,5.1,5.1)$ where x_0, y_0 are as before the initial prices for firm A and B respectively.

Application of the Runge-Kutta method gives the following price curves on Figure 3 and Figure 4 for $x(t), y(t)$:

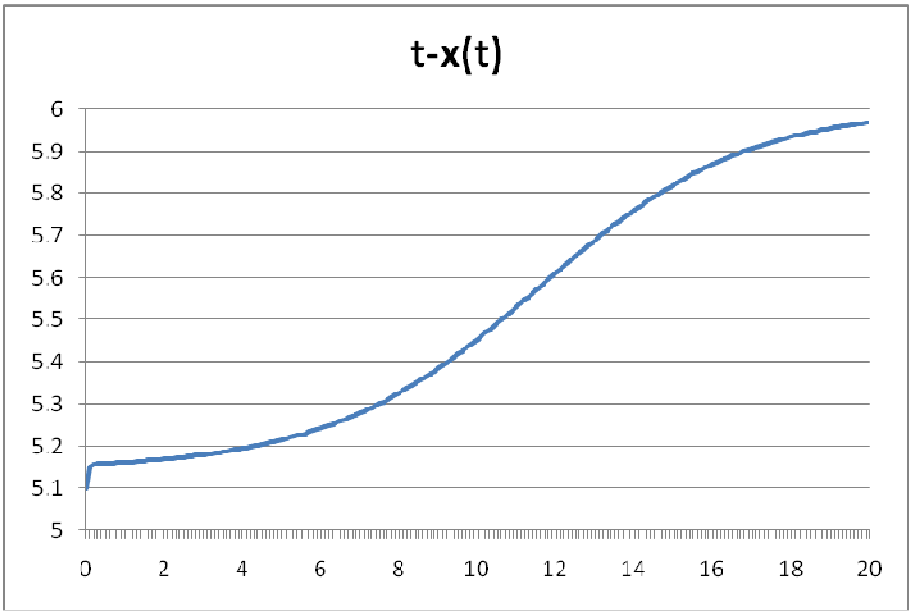

Figure 3. Price curve for x(t)

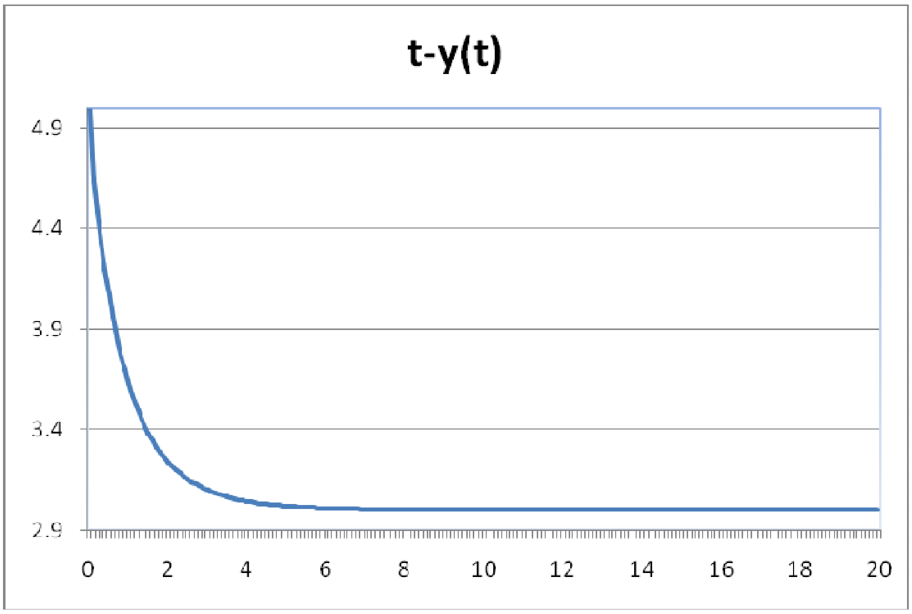

Figure 4. Price curve for y(t)

It is clear that as $m = 1, n = -5$ that A is fatigued from the price war while B is very much interested in pursuing a price war. As the above graph and our own intuition suggests this would cause a simple phenomena: A would increase its prices and B would decrease the same. As far as the price war is concerned this is all that would happen. The fact that this lowering of prices is not sustainable for B is beyond the scope of our price war model. However we may point out that such a situation occurs when one of the firms is facing bankruptcy and lowers its prices in an attempt to avoid it.

As in the previous case a further analysis of the observed equilibrium point $(6,3)$ may be carried out.

2.1 Model for k Firms

Our model is easily generalized to the case of k firms. If we let $x_i(t)$ denote the price of the ith firm at time t which can at most be $x_{i_{max}}$ and let the corresponding reaction, fatigue and grievance factors be a_i, b_i and c_i then the generalized model would consist of k simultaneous differential equations given by:

$$\frac{dx_i}{dt} = \left(1 - \frac{x_i}{x_{i_{max}}}\right)\left(-\sum_{j\neq i}\frac{a_j}{x_j} + b_i x_i - c_i\right) \text{where } i = 1,2,\dots,k$$

The generalized model can be solved on similar lines as in the special case $k = 2$.

3. MANAGERIAL IMPLICATIONS

Price war can be considered as a downward fall in prices from a collusive equilibrium level due to competition pressure. This reduction in prices by multiple firms in a market may not be economically beneficial for the individual firms. The only condition by which such huge price cuts can be made profitable for the firms is that the marginal cost should reduce faster than the price. It represents severe forms of competitive interplay in the market which causes both economic and non-economic costs [6]. The faster fall in marginal cost compared to price is a rare possibility which makes price war a very costly affair. Hence, price war is a strategy which firms try not to ignite unless there are other factors like financial condition of the firm or expectations.

Reasonable prediction for future prices is possible using this model. The firm wishing to start a price war can make an estimate of the competitors parameters (b, n, s) using market experience and statistical tools. With suitable planning one may decide on one's own parameters (a, m, r) and have information on the effect of the price war on prices in the near future. This may be used for strategic effect by managers of the firm.

4. CONCLUSIONS

We have modeled price movements of two firms which areindulging in price war using a pair of simultaneous differential equations. The resultant model can be solved numerically and analyzed. Such modeling can be solved numerically and analyzed using non-linear theory. It is found that generally prices tend to stabilize at some point above a minimum value during a price war.

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