Market Expansion Based Innovation Diffusion Modeling and Optimal Timing for Changing Management Strategy

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Summary: Up-grade and Up-date are the two important phenomenon’s that firms look up to improve and excel in the marketplace. In order to update their current marketing strategies firms have to undergo a decision making process for the optimal time to bring out that change. There are several factors like price declination, variation in advertising policies, packing and so on that can lend a positive impact on adoption process. But when to implement the strategy is a big challenge. In the current paper, we have used a methodical approach to see the impact of change in the adoption behavior after a certain time point called change point. The proposal has further been used to identify the optimal change point for the strategies adopted by the firm. Real life sales data set has been used to identify whether our finding is optimal; overestimated or underestimated.

Key words: Change point, dynamic market size, innovation diffusion model, update.

1. INTRODUCTION

The basic goal of marketing manager is to introduce new product or service into the market at a low cost. With the complexity and high competition in the market, management decisions may turn up crucial. The decisions regarding pricing, advertising, promotion strategies are all equally imperative for the success of new innovation. Generally, firms expect that a minimum level of sales might happen in the specified period of time. Several sales model based on Innovation Diffusion
Modeling exist in the literature which provide information regarding the market structure or the rate at which adoption occur [2, 4, 12]. These models may help in predicting the growth and stability of any firm in the market. Diffusion models being highly applied measure in determination of sales for different innovations and are used to understand the perspective and behavior of customers regarding product at different stages of the product’s life cycle. The four key elements of diffusion process as defined by Rogers [16] are: innovation, channels of communication, time and social system; Innovation is the creation of new product or service in the market which is diffused through channels of communication among the potential buyers (social system) over the time horizon.

Once the product is introduced in the market, the performance of that product can be better or worse than the expected value. The value of the product is estimated by the satisfaction of customers which is related to their income and the price of the product. The information regarding product spread through advertising by social media as well as by the inter-personal communication between the buyers. As more the influence of the satisfied buyers, more number of non-adopters will become the eventual adopters. Each individual tests the product to see whether it fulfill their expectation parameter (e.g. taste, ease of use, price, durability, etc.) or not. The positively motivated individual keeps a favorable attitude towards the product and also desire to buy the product. The rate of adoption is determined by diffusion of awareness about the product, which is controlled through advertising. Also, the rate of growth of the potential adopter population can be controlled through pricing of the product [18].

An innovation is adopted when top managers decide to go ahead with the new idea and allocate resources to it [15]. The changes in ideas/ during the implementation of innovation bring changes in the adoption of product. A sudden change in the adoption curve at some point in between the life cycle of the product is called change point. There are several factors that can cause change point in the sales. Some of them are: Promotional Efforts, Advertising strategies, Entry/exit of competitors from the market and Change in packaging of product and so on. It is not always possible that the expected adopters will remain same all the time. Those individuals who are not in queue of potential adopters in the past may adopt the product with the change in current price. For instance, due to the decrease in the price of Iphone 5S in October, 2014 i.e. during the product life cycle, the rate of adoption of Iphone 5S has increased tremendously [9]. Successful prediction of sales of the product can contribute significantly to a long term fiscal success of the company and it is an effective strategy to increase the adopters of the product. Marketing strategy is an important attribute in product sales growth. Mathematical tools have proved useful in understanding the expansion of market by predicting the sales of product and prescribing the best course of actions under known constraints. In the life-span of product, management up-date the product by changing product packaging, advertising and campaigning policies, introducing free gift vouchers etc. based on market needs, which will experience a drastic change in the sales of the product. Due to updating strategies, the adoption curve of product is likely to increase, as more number of individuals will get aware of it.

The objective of this paper is to propose a diffusion model under the assumption that the market size of the product will experience a radical increase in the number of adopters as and when the companies bring out some change in the strategy. In later part of paper, we have also discussed the related optimal time to make change in strategies that minimizes the total cost. The rest of the paper is organized as follows: In section 2 of this paper we give a brief of literature. In section 3 we define the notions and assumptions that are used in this study and in the later part of the section the proposed model is discussed. In Section 4, the model has been validated and results have been analyzed. The optimization cost model has been designed in Section 5 and in the subsequent Section 6, the numerical illustration has been provided to find the optimum change point time.
Managerial implications are shown in section 7 and at last, the paper has been concluded in section 8.

2. LITERATURE REVIEW

In literature, many related researchers have assumed that the market potential of a product remains constant over its entire life cycle. The most widely accepted innovation diffusion model in the literature is given by F.M. Bass in 1969. The Bass model [6] states that the probability that an individual will adopt the innovation, given that an individual has not yet adopted it, is linear with respect to previous adopters and has also assumed that the number of potential adopters remains constant throughout the life cycle of the product. Due to this assumption, the applicability of the model becomes limited. In real scenarios, market potential varies with changes in strategies of marketing overtime. A large number of modifications have been made by many researchers in past over the Bass model [6] to study the prevailing and variability market structure [4, 11, 14]. The nature of market size depends upon various sociological, economical, and political factors. Based on this paradigm, Sharif and Ramanathan [17] have proposed a model by considering the scenario of increasing market size. In 2004, Kapur et al. gave an alternative formulation of Bass model by considering the logistic function as rate of adoption. Mahajan and Muller [13] believed the now or at maturity rule. They conclude that it will be optimal to either initiate the improved product as soon as it is available or when enough sales have been mounted up for the previous product generation. Cohen et al. [7] assumed that the product can only be sold during a fixed window of time. Thus, delaying the product introduction time for further development can escort to a better product and will generate higher revenues over a period of time. Aggrawal et al. [2] found the optimal time of offering the advance generation product in the market under the cost constraints.

Very few of the researchers have focused on the change point concept in marketing innovation diffusion research. Jushan Bai [5] has analyzed the response of market interest rates to the changes in the discount rates. In 2007, Kapur et al. [12] incorporated the concept of change point in innovation diffusion model by using the varying rate of adoption per remaining adopters due to change in market strategy. Singh et al. [19] gave an innovation diffusion model using Stochastic Differential equation incorporates the changing behavior of adopters. The proposed research has incorporated the concept of change point as described in Kapur et al. [12] and model is based on the assumption that the size of market potential changes after a certain period of time. The diffusion model has been outlines by considering the market size as an exponential function of population growth after a fixed time. Thus, we attempt to construct a more precise growth model. It is also crucial to make changes at an appropriate time which should be cost effective. We have shown how the change in adoption and potential buyers takes place with change in marketing strategies. In the later part of the paper, by extending the diffusion model we found the optimal time at which strategies to be changed using cost minimize problem.

3. MODEL DEVELOPMENT

The following are the assumptions and notations that are used in the proposed model.

A. Assumptions
1. The target market increases exponentially with time after a particular time point.
2. The rate at which potential customers adopt the product is proportional to the remaining number total adopters.
3. All adoptions are independent and equally probable.
4. Buying behavior of the individual’s is characterized by the communication mode through which they receive the product information.
5. Rate of adoption may change at any time point.
6. Market strategies changes continually after a fixed time point $\tau$.
7. The rate of adoption follows logistic learning curve.

**B. Notations**

$m$ : Total number of potential adopters.

$b(t)$ : Adoption rate before the change time.

$b(t)$ : Adoption rate after the change time.

$N(t)$ : Expected number of adopters at time $t$.

$\tau$ : Time at which market strategies change i.e. change point time.

$\alpha$ : Rate at which new adoption incurs due to change in marketing.

$C_1$ : Cost of production of per unit before change point.

$C_2$ : Cost of production of per unit after change point.

$C_3$ : Promotional cost per unit time.

$C(t, \tau)$ : Total expenditure made by the firm for their product.

$C_B$ : Total budget allocated for product.

$N_0$ : Sales target after which market strategies can be changed.

**C. Model Formulation**

The study given by Kapur et al. [10] avoids the distinction between the innovators and imitators by considering the adoption rate to be logistic learning function. They used the following differential equation to represent the product diffusion process:

$$\frac{dN(t)}{dt} = b(t)[m - N(t)]$$  \hspace{1cm} (1)

Later, few researchers [4,17] have attempted to modify the bass model by incorporating the market increase phenomena. They considered that potential market does not remain constant throughout the life cycle of the product rather it changes with time. The mathematical interpretation of diffusion process given by Sharif and Ramanathan [17] is:

$$\frac{dN(t)}{dt} = b(t)[m(t) - N(t)]$$  \hspace{1cm} (2)

In the present paper, we induced the concept of change point in rate of adoption of an innovation. A sudden change in the adoption curve at some point $\tau$ in between the life cycle of the product may be due to certain socio-economic and political factors. Some of the factors are: promotional efforts, advertising strategies, entry/exit of competitors from the market and change in packaging of product and so on. Kapur et al. [12] gave the change point model by assuming that there is a fluctuation in an adoption rate before and after the change point. The total time duration of the product is divided into two intervals $[0, \tau]$ and $(\tau, t]$ to impose the change point concept in our study. We propose a
sales model incorporating the affect of changing market strategies in the social system. Integrating new policies in between life cycle of product can cause a drastic increase in adopters. The result shows the change in adoption rate before and after the change point time \( \tau \). We have assumed that the product adoption rate \( b(t) \) change at the time moment and it can be defined as:

\[
b(t) = \begin{cases} 
\frac{b_1}{1 + \beta_1 e^{-\beta_1 t}}, & 0 \leq t \leq \tau \\
\frac{b_2}{1 + \beta_2 e^{-\beta_2 t}}, & t > \tau 
\end{cases}
\]  

(3)

In general, before making any changes in the marketing strategies the number of potential adopters is assumed to be constant. On the other hand, changing policies (advertising, packing, government policies etc.) in market adds new adopters in the system. Considering change point time \( \tau \) we have also assumed that the market size changes exponentially into the system due change in market conditions/strategies, and the adopters are added by rate \( \alpha \). Therefore the market potential in both the cases can be designed as follows:

\[
m(t) = \begin{cases} 
m, & 0 \leq t \leq \tau \\
me^{\alpha t}, & t > \tau 
\end{cases}
\]  

(4)

Taking both the assumptions that are mentioned above into consideration, i.e. using (3) and (4), the adoption process of the proposed models shows the continuous flow of dynamic character of adopters. The corresponding differential equation for product adoption process can be build as follows:

\[
\frac{dN(t)}{dt} = \begin{cases} 
\frac{b_1}{1 + \beta_1 e^{-\beta_1 t}} \left\{ m - N(t) \right\}, & 0 \leq t \leq \tau \\
\frac{b_2}{1 + \beta_2 e^{-\beta_2 t}} \left\{ me^{\alpha t} - N(t) \right\}, & t > \tau 
\end{cases}
\]  

(5)

Using the initial condition at

\[
t = 0, \quad N(t) = 0 \quad \text{and} \quad t = \tau, \quad N(t) = N(\tau)
\]

On solving (5) with above mentioned initial conditions, we get the cumulative number of adopters at time \( t \) as follows:

\[
N(t) = \begin{cases} 
m \left\{ \frac{1 - e^{-\beta_1 t}}{1 + \beta_1 e^{-\beta_1 t}} \right\}, & 0 \leq t \leq \tau \\
\frac{m}{1 + \beta_2 e^{-\beta_2 t}} \left[ \frac{b_2 e^{-\beta_2 t}}{b_2 + \alpha} \left( e^{(b_1 + \alpha) t} - e^{(b_2 + \alpha) t} \right) + \frac{1 - e^{-\beta_1 t}}{1 + \beta_1 e^{-\beta_1 t}} \left( 1 + \beta_2 e^{-\beta_2 t} \right) e^{-\beta_2 (1 - t)} \right], & t > \tau 
\end{cases}
\]  

(6)

Equation (6) simply states the mean value function of proposed model with incorporating change in strategies at a certain time \( \tau \). Also it can be seen that the parameter \( \alpha \) increases the market size exponentially plays a significant role in estimation of cumulative number of adopters and the
parameters \( b_1 \) and \( b_2 \) (i.e. adoption rate before and after change point time \( \tau \)) plays a noteworthy change due to change in marketing policies.

4. DATA ANALYSIS

The proposed model has been validated on IBM System-in-use Generation –I (USA) [3] by using the position of change point time \( \tau \) as 8\textsuperscript{th} year. We have used SPSS tool based nonlinear least square method to estimate the parameters of the proposed model. The estimated parameter values of the proposed given by equation (6) and change point model given by Kapur et al. [12] are shown in Table 1. \( R^2 \) (Coefficient of determination) and MSE (Mean Squared Error) have been used as comparison criteria for model validation. Figure 1, shows the actual and the predict values of the data set by using the line graph where the values appear to be closely related which implies the moderately good fitness curve.

Table 1. Parameter estimates and Comparison Criteria

<table>
<thead>
<tr>
<th>Models</th>
<th>( m )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \alpha )</th>
<th>( R^2 )</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kapur et al. [12]</td>
<td>15890</td>
<td>0.537</td>
<td>0.620</td>
<td>16.734</td>
<td>32.854</td>
<td>-</td>
<td>0.999</td>
<td>12076</td>
</tr>
<tr>
<td>Proposed Model</td>
<td>15510</td>
<td>0.574</td>
<td>0.645</td>
<td>19.568</td>
<td>36.778</td>
<td>0.002</td>
<td>0.999</td>
<td>7946.451</td>
</tr>
</tbody>
</table>

![Figure 1. Goodness of fit curve](image)

5. OPTIMIZATION PROBLEM

In our research, we have developed the optimization problem whose objective is to minimize the total cost under the budgetary and minimum targeted sales in limited time as constraints. We take into account the two cost functions, namely, production cost and promotional cost. It has been observed that as more promotional efforts incurs by company at appropriate time, the more number
of potential adopters tend to purchase the product. Therefore, it is considered that promotional cost with time also increases. We are interested in understanding the optimal time to change in marketing strategies and various promotional efforts under the cost constraint. Cost function can be given as:

$$C(t, \tau) = C_1N(\tau) + C_2(N(t) - N(\tau)) + C_3\left(\frac{t}{1 - \alpha}\right)$$  \hspace{1cm} (7)$$

Throughout the analysis, we assume that \( C_2 > C_1 \). As the promotional efforts have been made at time \( \tau \) to increase sales will also increase the cost of production per unit. Here, time point \( \tau \) acts as a decision variable that has to be minimized. In (7), the cost function comprises of: the production cost before and after change point time \( \tau \) (i.e. first and second component) and the third component defines the cost of promotion efforts per unit time. We are taking \( t/ (1 - \alpha) \) as a time point where the time span is significantly large so more and more adoptions take place. Here, \( t \) is the time in which company is planning to analyze the sales trend. Due to rapid development and pace of technology, the product has fixed life span. Also, competitor’s product and obsolescence leads to limited life window in which marketers have to absorb the sales of the total market potential. Therefore, in our study we assume the constant life cycle of the product. In this time duration, we have tried to find out the optimal time point where management brings changes in the marketing strategies following minimal cost of the product.

To determine the optimal change point time \( \tau^* \), the optimization problem can be defined as:

Min. \( C(t, \tau) = C_1N(\tau) + C_2(N(t) - N(\tau)) + C_3\left(\frac{t}{1 - \alpha}\right) \)

Subject to constraints

\( C(t, \tau) \leq C_B \)
\( N(\tau) \geq N_0 \)  \hspace{1cm} (8)

Here we consider the total cost should be less than the budgetary cost i.e. the promotional cost and production cost should be bounded to make change in marketing strategies and also optimal decision is to hold till the achievement of minimum sales target. The above formulated equation (8) is the set of optimization problem which helps the firm in determining the minimum total cost of product under budgetary and sales constraint.

6. NUMERICAL ILLUSTRATION

The purpose of this study is get into insight of various corners of optimization techniques and also to study the effect of change in marketing strategies effect on cost and adoption behavior of adopters. We have considered the data set of IBM system in use of generation I from Kapur et al. [12] where they have used the change point time \( \tau = 8^{th} \) year after introduction of product in the market. To solve the optimization problem, we use the parameter values of equation (6) found in table 1 i.e. \( m = 15510, \ b_1 = 0.574, \ b_2 = 0.645, \ \beta_1 = 19.568, \ \beta_2 = 36.778, \ \alpha = 0.002 \) and other base values considered in this paper are: \( C_B = \text{Rs 20,000}; \ C_1 = \text{Rs 10}; \ C_2 = \text{Rs 20}; \ C_3 = \text{Rs 8}; \) and \( N_0 = 10,000 \) i.e. a minimum sales of 10,000 should be achieved before introducing any change in market place. In our study we have assumed that the product has a fixed window i.e. \( t \) is fixed which we have taken to be 21 here (in years). According to firm’s perspective, it is important to set a
minimum target sale to achieve prior make any change and a maximum limit of budget within which all costs can be paid. Using optimization solver (LINGO) to solve the equation (8) with all above mentioned values, we find the optimal change time \( \tau^* = 12.32 \) with minimum cost \( C(21,12.32) = Rs 172539.4 \). Therefore, if the firm change their strategies at time 12.32\(^{th}\) year will be cost effective and can reduced cost significantly then the change point considered by Kapur et al. [12] i.e. 8\(^{th}\) year where cost \( C(21,8) = Rs 191188 \).

7. MANAGERIAL IMPLICATIONS

Mounting competition and other complexities in market make it imperative for managers to better understand and target the market cost-effectively. Managers need to examine the consumption pattern of customers so that they can plan their strategies accordingly. Every management have some questions in their mind before introducing any change in market i.e. when to change? For whom to change? At what cost to change? The present study describes the several factors that affect the purchasing decision of the customers. It helps in implementation of marketing plans in between the life cycle of the new product with minimum cost to the company. The proposed model helps the managers to find an appropriate time to change meaningful and effective marketing programs into action.

8. CONCLUSIONS

The company's main task is to creatively communicate their message to large number of customers and also keep in mind the cost effectiveness. After diffusion of the product, firms need to modify in their traditional structure of the product to sustain in the world of competition. Thus, they have to distinguish themselves from their competitors through the forces of change in their strategies. The changes must be inculcated on time else it leads to monetary loss to firms in various forms. As early changes in strategies will increase the promotional cost whereas a little delay in making changes can lose the potential buyers to other competitors. To make decisions regarding the changes in market strategies at appropriate time, firms need to estimate this time with maximum validity. So, in this paper we present a dynamic change point model which incorporates the change in adoption behavior of the customers by altering policy or marketing efforts in strategies after a certain point of time. Additionally, a methodical approach is also presented to estimate the optimal cost and optimal change point time for the product. Numerical illustration has been shown which proves that finding the time to bring out the change in strategy is more cost effective rather than taking it randomly. Also, our analysis has find out that the change in positive scenario of marketing strategies attracts more number of customers to purchase the product.

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