CDQM, An Int. J., Volume 17, Number 3, September 2014, pp. 35-41

COMMUNICATIONS
IN
DEPENDABILITY AND
QUALITY
MANAGEMENT
An International Journal

UDC 629.7.051.8:519.711
COBISS.SR-ID 213391116

Methods of Analysis of Maximum Permissible Risk At Automatic Landing of Airplanes

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accepted August 20, 2014

Summary
Now the concept of acceptable risk has taken a stable position in problems of safety management. Besides the new *direction - risk management has started to develop. As the risk has a probability character, all methods of the impact analysis of catastrophic failures transmitted from a reliability theory to operation of systems in the theory of risk management. These methods have preventive character and ensure admissible risk rate at a design stage. However in a number of problems of potentially dangerous engineering systems safety management security and confirmation to requests of the maximum permissible risk rate probability up to* $10^{-6}\ldots10^{-8}$ *is required. Such requests can be confirmed only with the use of statistical mathematical simulation and tools of modern computer facilities. However the classical statistical theory does not decide problems with midget probabilities. In existing normative documents any evaluation procedures of such probabilities are also not regulated. Therefore research and creation of methods and algorithms of "measurement" of midget risk rates is a new actual problem. In the article perspective directions of a solution of this problem on an example of airplanes automatic landing systems correspondence confirmation to standards of the air validity are revealed.*

Key words: *Acceptable, maximum permissible risk, sudden and parametric failures, probability distribution functions, probability methods of the analysis of reliability, statistical estimation methods.*

1. INTRODUCTION

Requirements to precision characteristics of civil airplanes automatic landing are divided on two groups: requirements to comfortable landing and requirements to safe landing. According to the modern concept of acceptable risk these requirements are set in the form of admissible values precision characteristics with the indication of probability of an output of limits of the tolerance (risk). Thus to the first group of requirements corresponds acceptable risk of the order 0,05 (Table 1), and to the second - maximum permissible risk of the order 10^{-6} (Table 2). The estimation of conformity of acceptable risk to standards of flight validity does not represent any difficulties and at certification of the airplane can be conducted according to flight tests, but the estimation of maximum acceptable risk represents the new task of mathematical statistics and can be solved at a stage of mathematical simulation which according to normative documents should be recognized legitimate at certification of an airplane.

Table 1. Requirements to precision characteristics of the automatic landing system of airplanes Il-96 and Tu-204

Condition	The Category of a condition	Requirements
Landing	The Landing approach	At least 95 % of landing approaches should meet the following requirements: 1) With H = 150 m the landing approach should be completed without loss of functions by system. 2) Between H = 150 m and a flare initiation the velocity of flight should be withstood within the limits of +9, 3 km / h rather Vag without taking into account the fast velocity modifications caused by turbulence. 3) Below H = 90 m of deviation concerning equisignal zones should not exceed: - For automatic control: 0,0762 RGM on a glide path up to H = 30 m; 0,0206 RGM on a course zone up to H = 0; - For director control: 0,1 RGM on a glide path up to H = 60 m; 0,026 on a course zone up to H = 60 m $P\left\{\begin{array}{l}\text{conservation functions at } H<150m\\ \lvert\Delta V_{for}\rvert \le 9{,}3km/h \text{ at}\\ H=150m \div H_n;\\ \lvert\varepsilon\rvert \le \varepsilon_{add} \text{ at } H \le 90m\end{array}\right\} \ge 0{,}95$
	Touchdown	1) Longitudinal dispersion of tangency points concerning a nominal at symmetrical allocation with P = 0, 95 should not be more than 225 m\ $P\{\lvert\Delta L_{land}\rvert \le 225m\} \ge 0{,}95$

		2) Side deviation with P = 0, 95 should not exceed 8,2 m $P\{\lvert Z_{land}\rvert \le 8{,}2m\} \ge 0{,}95$
	Run	Automatic control postlanding up to a velocity of 75 km / p should implement run with the performances set for touchdown $P\{\lvert Z(t)\rvert \le 8{,}2m\} \ge 0{,}95$

Table 2. Requests of Uniform West-European airworthiness standards to maximum permissible risk

Criteria of performances	Probabilities of excess
Touchdown in a point arranged on longitudinal distance from a threshold of runway less than 200 ft.	10^{-6}
Tangency of runway in a point arranged outside lights of illumination of a zone of touchdown, i.e. on longitudinal distance from a threshold of the runway, exceeding 3000 ft.	10^{-6}
Tangency of runway by a side wheel of a landing gear in a point arranged on side distance from a runway center line, exceeding 70 ft. with the supposition, that the breadth of runway is 150 ft.	10^{-6}
The vertical landing velocity corresponding the limitation on strength.	10^{-6}
Bank angle at which the tip of a wing touches runway before wheels of a landing gear.	10^{-8}
Lateral velocity or the yaw angle corresponding the limitation on strength.	10^{-6}

2. ANALYSIS OF MAXIMUM PERMISSIBLE RISK AT AUTOMATIC LANDING OF AIRPLANES

Under the theory of ordinal statisticians between minimum $x_{(1)}$ and maximum $x_{(n)}$ values of sample of a volume *n* there is a share of distribution R of the examined characteristic x, determined by a relation:

$$nR^{n-1} - (n-1)R^{n} = 1-\gamma,$$

Where g – is a fiducial probability.

The necessary sample size *n*, ensuring a share, distributions *R*, cut off $x_{(n)}$ for $R = 0{,}999999$ ($1-R = 10^{-6}$) makes $n \approx 2{,}3\cdot 10^{6}$, and the share taken between $x_{(1)}$ and $x_{(n)}$ demands - $n \approx 3{,}9\cdot 10^{6}$ with a fiducial probability $\gamma = 0{,}9$.

Considering that at optimization of control laws of automatic airplane landing over 10 adjustings are made, holding of statistical simulation of such volumes of data seems inexpedient as on temporal and on economic limitations. Therefore there is a necessity of development of the mathematical means allowing by results of the limited volume simulation to conduct the prognosis of correspondence/discrepancy of precision characteristics to the shown demands and thus to justify the necessity of corrective actions at early stages of simulation. In mathematical statement the given problem of proximate analysis of automatic landing precision characteristics is reduced to a primal

problem of unobservable "tails" of distributions. Research of possible paths of a solution of the task in view began with the analysis of possibilities of known methods.
Classical mathematical statistics does not ensure solving of problems, connected with the midget valued risks. Nevertheless the research of possible paths of the task solution started with the analysis of capabilities of known methods.

The reviewed task of prediction can be solved only by parametric methods by exposition of empirical laws of distribution by some analytical associations, initial attempt of such exposition rested upon the use of known sets of distributions of Pearson and Johnson. For of some precision characteristics at a tangency of a run-way approximation by these methods has yielded satisfactory outcomes [2].

In Table 3 seven types of Pearson’s allocation sets in standard denotations are presented.

Table 3. The set of Pearson's allocations

Type	The Equation	The Zero reference datum for x	The Define area
I β - allocation	$y = y_0\left(1+\frac{x}{a_1}\right)^{m_1}\left(1-\frac{x}{a_2}\right)^{m_2}$	Mode	$-a_1 < x < a_2$
II β - allocation	$y = y_0\left(1-\frac{x^2}{a^2}\right)^{m}$	Mode (average)	$-a < x < a$
III γ - allocation	$y = y_0 e^{-\gamma x}\left(1+\frac{x}{a}\right)^{\gamma a}$	Mode	$-a < x < \infty$
IV Special allocation	$y = y_0 e^{-\gamma a rctg\frac{x}{a}}\left(1+\frac{x^2}{a^2}\right)^{-m}$	Average $+\frac{\gamma a}{2m-2}$	$-\infty < x < \infty$
V inverse γ - allocation	$y = y_0 e^{-\gamma/x} x^{-p}$	Has started a curve	$0 < x < \infty$
VI inverse β - allocation	$y = y_0 (x-a)^{q_2}\, x^{-q_1}$	Offset point on $\lvert a \rvert$ from the beginning of a curve	$a < x < \infty$
VII Student's distribution	$y = y_0\left(1+\frac{x^2}{a^2}\right)^{-m}$	Average, mode	$-\infty < x < \infty$

In Table 4 three types of the set of Johnson’s allocations [2] are presented.

Table 4. The set of Johnson's allocations

Type	The Equation	The Define area
S_L	$y = \gamma + \zeta \ln\left(\frac{x-\varepsilon}{\lambda}\right)$	$x > \varepsilon$
S_B	$y = \gamma + \zeta \ln\left(\frac{x-\varepsilon}{\lambda+\varepsilon-x}\right)$	$\varepsilon \le x \le \varepsilon + \lambda$
S_n	$y = \gamma + \zeta \ln\left(z + \sqrt{z^2+1}\right),\ z = \frac{x-\varepsilon}{\lambda}$	$-\infty \le x \le \infty$

Approximation by known distributions of sets of Pearson and Johnson is rather attractive, because the selection of approximating distributions is based on use of the empirical moments of distributions precision characteristics which already at $n = 3*10^5$ have no statistical scatter, Table 5.

Table 5. Results of approximation of probability distributions of precision characteristics of automatic landing of airplane AN-148 with allocations of Pearson and Johnson [3]

Parameter of landing	Type of allocation	Prognosis values on n=10^6	Experimental values on n=10^6
Angle of bank	VII type of Pearson's Allocation	± 5.05°	4,90°
Angle of drift	-	± 4.64°	4,75°
Lateral deviation	-	± 8.73м	7,79 m
Vertical velocity	S_n Johnson's allocation	3,48 m\s	3,48 m\s

So in Table 6 the values of the empirical moments of distance of a tangency of an airplane AN-148 for various sample sizes [4] are given.

Table 6. The empirical moments of distance of a tangency

Sample size n	Expected value, m	Standard deviation	Asymmetry	Kurtosis
$3*10^5$	400,713	40,374	0,02	4,72
$6*10^5$	400,72	40,317	0,02	4,73
10^6	400,689	40,339	0,022	4,65

However, some airplanes had the "complicated" characteristics limiting safety of landing for which the given methods could not discover a possible solution (for example, for a vertical velocity of airplane Il-96, for distance of a tangency of airplane An-148).

The further researches have shown, that, for example, for distance of a tangency a "discord" is characteristic when owing to nonlinearity of a control system there is a modification of probability law from normal for a central part to unknown for "tail" parts of the distribution. In these conditions the exposition of the empirical law of distribution in all range of a modification random precision characteristics by any one "pure" distribution is impossible.

The researches of a feasibility of a mixture of distributions have been carried out (known in practical solving of similar tasks in the USA and Japan) [2, 4]:

$$F(x) = \sum_{i=1}^{K} \alpha_i F_i(x, m_i, \sigma_i),$$

Where:
K - is a number of components of a mixture,
α_i - weighting coefficients ($\sum_{i=1}^{K} \alpha_i = 1$),
F_i - making mixtures,
and also possibilities of exposition of the limited final (initial) part of distribution by truncated distributions (Pareto, truncated normal) [4].

On the basis of comparison of these methods the sufficiency of these restricted final (initial) parts of a full empirical distribution of probabilities for the purposes of an extrapolation on unobservable "tails" of statistical exposition is shown. Thus both of the considered methods yield approximately equal outcomes (Table 7), therefore as a basic approach more simple approximation by distribution of Pareto has been selected.

Table 7. Outcomes of forecasting (prediction) of distance of a tangency

Number Implementations	The Method Approximatings	Forecasting result	
$3*10^5$	Pareto	839,673	Experimental value on 1 million implementations - 814,9 m
	Mixtures	857,746	
$6*10^5$	Pareto	830,435	
	Mixtures	848,35	
10^6	Pareto	815,865	
	Mixtures	829,842	

Let's reveal the algorithm of calculation that uses of distribution of Pareto. Distribution of Pareto looks like:

$F_{Pareto}(x) = 1 - \left(\frac{C_0}{x}\right)^{\alpha}$, At $x \ge C_0$.

The estimation by the method of moments of the unique parameter α is the estimation $\hat{\alpha} = 1 + \sqrt{1 + \left(\frac{1}{\hat{V}}\right)^2}$, where an estimation of factor of a variation is $\hat{V} = \frac{S}{\bar{x}}; \bar{x} = \frac{1}{n_{yc}} \sum_{i=1}^{n_{yc}} x_i; S^2 = \frac{1}{n_{yc} - 1} \sum_{i=1}^{n_{yc}} \left(x_i - \bar{x}\right)^2$.

The extent of truncation is $F_{truncated} = \frac{n_{sample} - n_{truncated}}{n_{sample} + 1}$,

Where:

$n_{truncated}$ - number of measurements in an examined tail part.

The coordination of distribution of Pareto with the truncated initial distribution is made by formula $F = F_{Pareto}\left(1 - F_{truncated}\right) + F_{truncated}$.

Prediction on $1 \cdot 10^6$ is carried out at substitution $F = 0{,}999999$.

Possibility of using of Pareto distribution for prediction of all precision characteristics of landing (table 8) has been examined.

Table 8. Outcomes of prediction for AN-148 airplane automatic landing

Parameter of a tangency	Exp. values on $6*10^5$	Exp. values on 10^6	Prognosis on 10^6	Results of flight tests
Dist. of tangency	802,39 m	814,9 m	815,865 m	684,8 m
Vertical velocity	3,37 m\s	3,48 m\s	3,48 m\s	1,15 m\s
Angle of bank	4,89°	4,9°	4,98°	3,1°
Lateral deviation	7,32 m	7,97 m	7,69 m	5,12 m
Angle of a drift	4,75°	4,75°	4,83°	2,26°

It is natural, that outcomes of the simulation conducted in a full range of random disturbing factors, give the worst values, than outcomes of the limited volume of the flight tests conducted in conditions close to nominal.

The conducted statistical simulation of automatic landing of airplane AN-148 confirms correspondence of precision characteristic of landing to standards of the flight validity and has been used at deriving the certificate of the flight validity of the airplane. As a result of the conducted researches the universal technique of proximate analysis has been generated.

3. FINAL NOTICE

Deviation of "tails" of laws of allocation from normal is characteristic for a lot of tasks of a quality management and safety (research of concentration of harmful additives in water, key cards at a disorder of technological processes, a method of a quality management “six sigmas”). Therefore the offered approach can discover a wider circulation, than for tasks of aircraft construction.

ACKNOWLEDGEMENT

The article is prepared at financial support of the Ministry of Education and Science of Russian Federation within the limits of the high-schools state research (theme №1636/14).

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