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The Method of Extrapolation on Unobservable "Tails" of Distributions of Airplanes' Automatic Landing Safety Indicators Probabilities

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Summary
On the basis of the analysis of empirical functions of allocations of the basic characteristic determining safety of automatic landing of airplanes, the original method of their mathematical exposition with use of piecewise approximation of their tail parts is offered. The method allows to conduct an extrapolation on unobservable tails of allocations and to carry out a rapid analysis of an estimation of conformity on the limited volume of statistical tests at improvement of automatic landing systems.

Key words: *The automatic landing system of an airplane, demands to safety, approximation of probability law.*

1. INTRODUCTION

At certification of automatic landing systems the estimation of a level of safety of automatic touchdown of an airplane is required. This problem can be solved, if demands to precision to characteristics of a condition of automatic landing on probability of intolerable errors of control are satisfied.

It should be shown, that characteristics of touchdown are those, that the exit for limits of any of the limitations given in Table 1 is quite an improbable event if variable factors are subject to distribution laws expected for them and also when one of them receives maximum permissible value while the remaining are subject to desired laws of allocation.

Table 1. Demands to maximum permissible characteristics of automatic landing systems of some trunk-route airplanes

Characteristics	Dispersion of tangency points along a surface of runway (distance of a tangency)	The Lateral deviation of a primary strut of a landing gear from a center line of runway	The Vertical velocity (modulo)	Pitch angle	Bank angle	The Slip angle
Probabilities of overflow 1-R_{apr} (on the average)	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-8}	10^{-6}
Probabilities of overflow 1-R_{apr} (in marginal)	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-7}	10^{-5}

Demands should be confirmed with the given confidential probability. In mathematical statement the problem of affirming of demands in such probabilistic form is reduced to a problem of build-up of a tolerance interval:

$$P\left\{\int_A^B f(x)dx \geq R\text{зад}\right\} = \gamma,$$

Where $f(x)$ - an elementary probability law of the explored characteristic [1].

Discriminate parametric and nonparametric tolerance intervals. A nonparametric tolerance interval, not dependent on a distribution law, gain by selection:

$A = X_{(r)}; \quad B = X_{(s)}$, Where $X_{(r)}$, $X_{(s)}$ - ordinal statistics.

For universality of this interval it is necessary "to pay" in great volume of the sample securing the given share of allocation Rзад, being between r-th and s-th values of the ranked measurings of the characteristic.

So the one-sided nonparametric tolerance interval is determined by expression $R^n_{\text{зад}} \leq 1-\gamma$, whence at Rзад = 0,999999; = 0,9 it is had n = 2302585.

Considering, that at improvement of control laws it is necessary to make some tens of iterations, even at capabilities of the modern computing machinery to implement such volume of mathematical model operation inconveniently. Making the mathematical method for a rapid analysis with use of more "economical" parametric tolerance interval is necessary. Really, at a known kind of a density function of probability $f(x)$ on the basis of a statistical modelling it is enough to determine estimations of an ensemble average m and variances σ^2 which as practice of model operation has shown, already at n = 30000 practically have no statistical dispersion. Then $A = m + k\sigma$; $B = m - k\sigma$, where the quantile $k = k(R\text{зад})$ - depends on a kind of allocation.

Knowing a kind and parameters of a distribution law of the explored characteristic, it is easy to execute an extrapolation on unobservable tails of allocations. However in practice of unknown persons are not only the moments, but also a kind of a distribution law.

Matching of a theoretical kind of a distribution law on experimental data is most a challenge of mathematical statistics. The methods grounded on use of sets of allocations of the Pearson (Pearson K.), the Johnson (Johnson N. L.) and some series [2, 3] are most known. Cramer's research (Cramer H.) [2] have shown that the use of application of series for the exposition of "tails" of allocations is inexpedient, since can lead to obtaining of the negative probabilities.
Build-up of allocations from sets of Pearson and Johnson is grounded on use of the first four moments and allows to gain matching densities of probabilities, thus the subsequent integration necessary for determination of a tolerance interval, is related to some computational difficulties.

Naturally there is a problem, whether it is impossible to select analytical relations at once to a distribution law? Such approach has been developed by I.W. Burr and M.A. Hatke and based upon equaling of the cumulative moments to the theoretical values expressed in terms of parameters of allocations [3]. The method is complex enough and has not discovered operational use. Other method of "adjustment" of analytical relations to the experimental data, developed by H.S. Sichel and consisting in equaling the theoretical and empirical probabilistic moments, also has not discovered wide application.

All the listed methods allow to approximate experimental data such mathematical expressions which are spread to unobservable values of random quantities. Other approach at which "adjustment" implements only in that interval where observed data is disposed is possible also, thus some "truncated" allocations are used.

The original approach offered in the article represents evolution of last idea with the purpose of an extrapolation of the gained results on unobservable tails of allocations.

2. THE LOGICAL JUSTIFICATION OF THE APPROACH

The logical justification of the given approach consists in the following:

1. The Input information is formed by empirical functions of allocations. Function of a normal distribution law in the matching gauge represents a straight line (figures 1-5). For more detailed pictorial viewing of the so-called “tails” it is desirable to conduct a non-linear tension along an axis of probability, i.e. to map an interval of probabilities (0,1) in an interval $(-\infty,+\infty)$. Operation of a tension of such type can be executed by many expedients. If for operation of a tension to select function, inverse to a standard normal distribution then in case the empirical distribution matches to a normal distribution, the chart of an empirical distribution will have a straight-line appearance. Therefore on charts of distribution functions of explored characteristics of automatic landing diversions from normality in tails of allocations (unlike bar graphs of elementary probability laws - figures 6-10) are visible. The analysis of functions of allocations is applied also at model operation of automatic landing an airplane Boeing 757/767 [4].

2. From viewing distribution functions follows, that in a central part (probability 0,01÷0,99) good coincidence the normal law is observed, and diversions from normality begin in tails of allocations. This fact can be given an engineering justification: the automatic landing system of an airplane is non-linear and at greater diversions of random factors effecting it from their mean these nonlinearities are manifested also distribution laws accordingly vary. Thus, in all range of fluctuation of characteristics unlike classical approaches it is impossible to use any one distribution law, but only some combination of various laws.

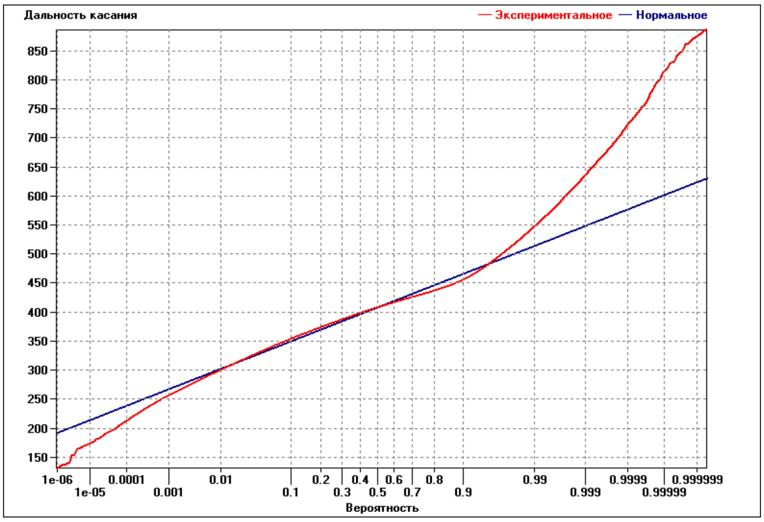

Figure 1. The distribution function of distance of tangency Dкас ($N=2{,}3\cdot 10^6$)

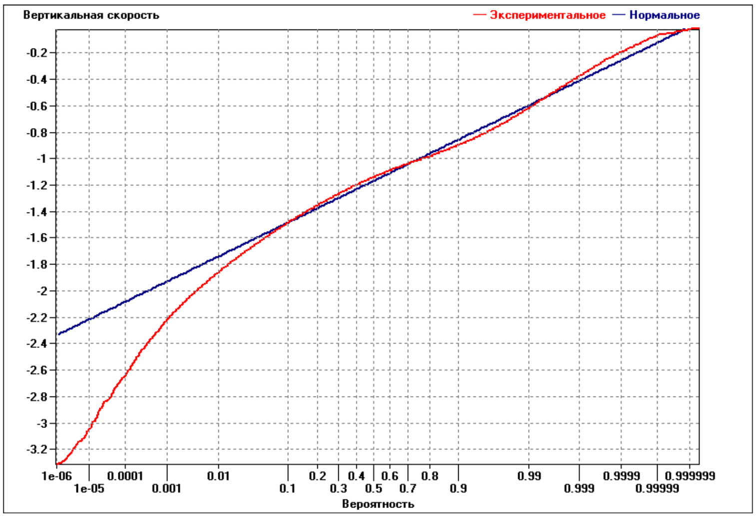

Figure 2 - the Distribution function of a vertical velocity (Vy) on a tangency ($N=2{,}3\cdot 10^6$)

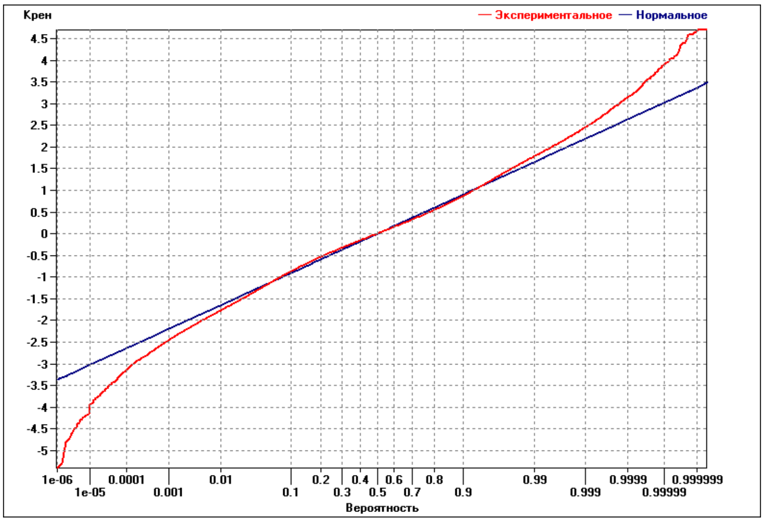

Figure 3. The distribution function of bank angle (Υ) on a tangency ($N=2{,}3\cdot 10^6$)

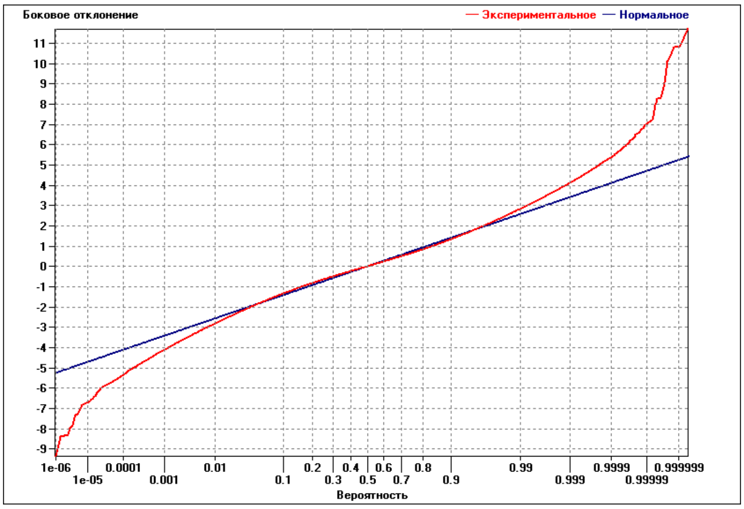

Figure 4. The distribution function of a lateral deviation (Z) on a tangency ($N=2{,}3\cdot10^6$)

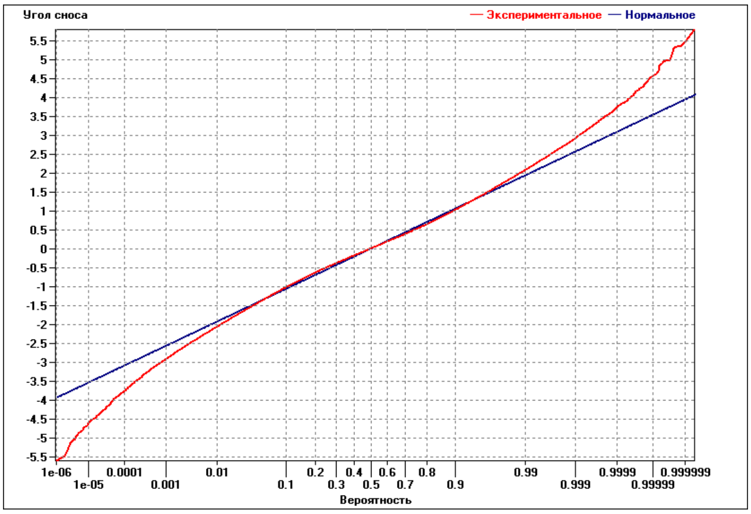

Figure 5. The distribution function of an angle of drift (YS) on a tangency $N=2{,}3\cdot10^6$)

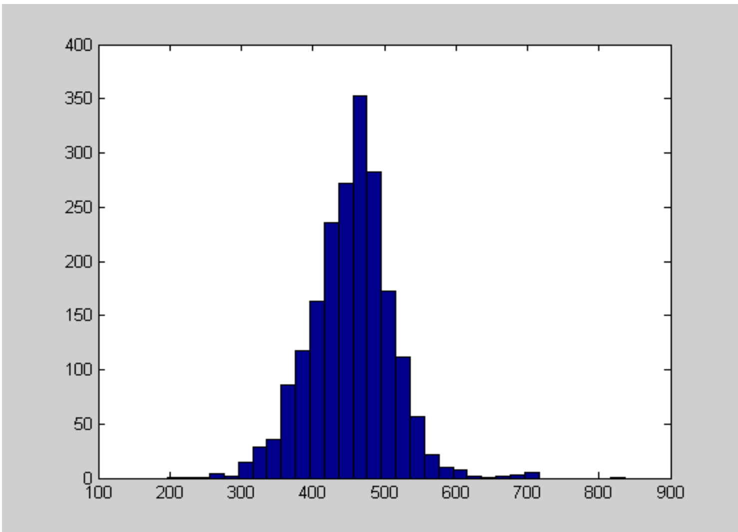

Figure 6. The bar graph of distances of a tangency (meters) by results of model operation of automatic landing AN-148 on a Monte-Carlo method ($N=2{,}3\cdot10^6$)

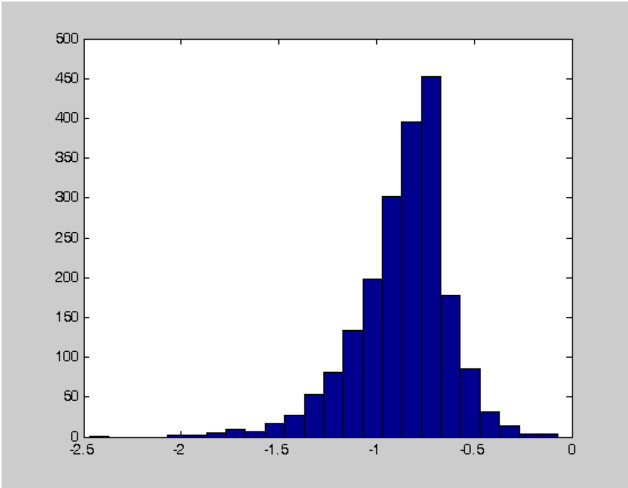

Figure 7. The bar graph of a vertical velocity (m\s) by results of model operation of automatic landing AN-148 on a Monte-Carlo method ($N=2,3{\cdot}10^6$)

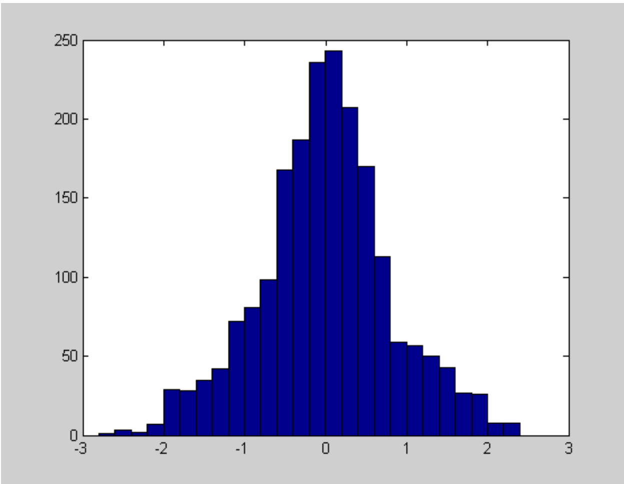

Figure 8. The bar graph of bank angle (deg) by results of model operation of automatic landing AN-148 on a Monte-Carlo method ($N=2,3{\cdot}10^6$)

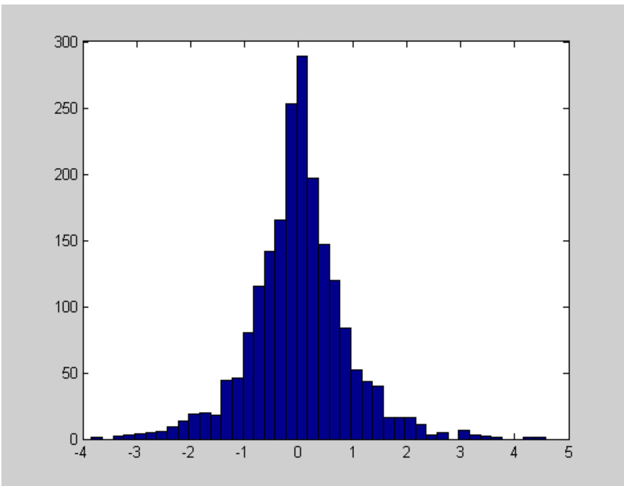

Figure 9. The bar graph of a lateral deviation (meters) by results of model operation of automatic landing AN-148 on a Monte-Carlo method ($N=2,3{\cdot}10^6$)

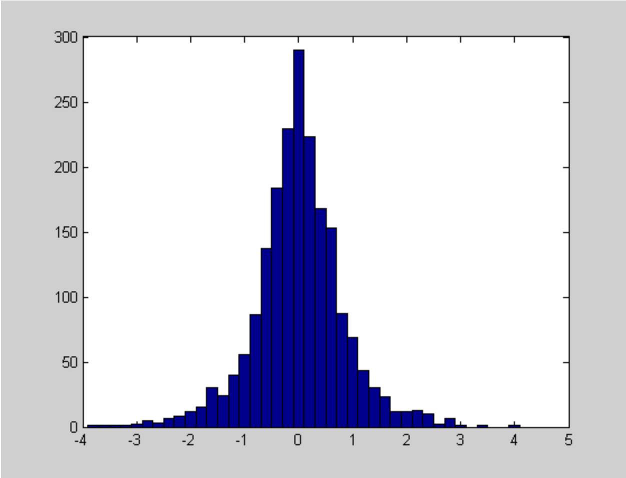

Figure 10. The bar graph of an angle of drift (deg) by results of model operation of automatic landing AN-148 (N=2,3·10^{6}

3. For approximation of tails of allocation the various laws limited at the left (if necessary limitations modules on the right are considered) can be selected. Thus matching of this or that law is stringently individual for each separate characteristic and each type of aircraft [5].

4. From the theory of interpolation it is known, that the interval of interpolation less, the coincidence data points is better. As for the solution of a problem of an extrapolation on unobservable tails of allocations us the exposition of all distribution function but only initial or its final part, selection of a volume of the ranked sample necessary for approximation needs to be yielded from a requirement of the compromise between precision of statistical estimations of the moments of a selected parent distribution and precision of interpolation does not interest.

By way of illustration results of approximation of the right tail of a distribution function of distance of a tangency are given in the offered approach on the basis of mate of a normal distribution with allocation of Pareto [6,7].

3. USE OF ALLOCATION OF PARETO

Allocation of Pareto, features allocation of a random quantity, greater (smaller) some fixed value C_0.

Allocation of Pareto looks like:

$$F(x) = 1 - \left(\frac{C_0}{x}\right)^a$$

At $x \geq C_0$ with an ensemble average

$$M[x] = \frac{\alpha}{\alpha - 1} C_0$$

and a variance

$$D[x] = \frac{\alpha}{(\alpha-1)^2(\alpha-2)} C_0^2$$

Estimation a method of moments of single parameter α is the estimation:

$$\hat{\alpha} = 1 + \sqrt{1 + \left(\frac{1}{\overline{V}}\right)^2}$$

Gained by equaling of the theoretical and empirical moments, where an estimation of coefficient of a variation:

$$\overline{V} = \frac{S}{\overline{x}}; \ \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ ; \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$$

The degree of truncation is equal:

$$F_{ус} = \frac{n_{выборки} - n_{ус}}{n_{выборки} + 1} \ \text{at} \quad F_{Парето} = \frac{n_{ус}}{n_{ус} + 1}$$

where

$n_{ус}$ - number of measurings in an explored tail part.

The coordination of allocation of Pareto with the truncated initial allocation is yielded by formula:

$$F = F_{Парето}(1 - F_{ус}) + F_{ус} \ .$$

In Table 2 results of approximation of a unobservable "tail" part of one of versions of distance of a tangency of an airplane AN-148 of a flight strip are given.

Table 2. Results of approximation of distance of a tangency allocation of Pareto ($n_{sample} = 3 \cdot 10^5$ *)*

$n_{ус}$	c_0	$\overline{x}$	S	$\hat{\alpha}$	$D_{\max}$ calc.	$D_{экс.}$	$D_{forecast}$ на $n = 10^6$	$D_{\exp.}$ $n = 10^6$
141	730 м.	763,44 м.	31,6 м.	25,2	890 м.	869,9 м.	932 м.	889,6 м.
20	800 м.	828,765 м.	22,87 м.	37,25	867,96 м.	869,9 м.	896,9 м.	889,6 м.

4. CONCLUSION

Thus, the use of Pareto allocation for interpolation of observable and an extrapolation unobservable "tail" parts of a probability distribution is one of the most difficult touchdown safety indicators and yields precise enough results. It can be recommended for other safety indicators with the purpose of making of the unified conformity affirmation procedure of maximum permissible risk at automatic landing of airplanes to standards of flight validity.

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