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# Methods for Express Analysis Safety Indicators Aircraft Automatic Landing At the Stage of Mathematical Modeling

***L. N. Alexandrovskaya*[1], *O. B. Kerber*[2], *A. E. Ardalionova*[2]**
*and* ***A.V. Kirillin*[1]**

[1] *Russian State Technological University Named After K. E. Tsiolkovsky - MATI, Faculty, Flight Vehicles Tests, Bernikovskaja Emb, 14-2, Moscow, Russia*
*E-mail: kirillinav@mati.ru*
[2] *Joint Stock Company Moscow Institute of Electromechanics and Automation, Aviatsionniy Lane, 5, Moscow, Russia*
*E-Mail: kirillinav@mati.ru*

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***Summary***
*The paper contains a consideration of the specifics, arising when operating on large sample volumes (n = 100 000 ÷ 1 000 000), obtained during statistical simulation of ICAO Cat. III aircraft automatic landing safety assessment problems and gives out recommendations on the necessary number of statistical tests justification and the probabilistic safety figures estimation procedures building.*

***Key words:*** *Automatic landing safety, mathematical simulation, mixture distribution, pareto distribution, probabilistic methods, risk assessment, spline-approximation.*

Air safety is the priority parameter of the air vehicles quality. Especially rigid requirements are applied to the performance, affecting safety in the process of the aircraft automatic landing. So Russian and foreign regulatory documents [1, 2, 3] specify performance requirements for the Category III automatic landing accuracy.

It is necessary to show that landing characteristics are such that the exceedance of any limit given below is improbable (P=10-5 ÷ 10-8) if variable factors follow the expected distribution laws (average) and also when one of them takes maximum acceptable value (limited) while all the others follow the expected distribution laws (Table 1).

Thus these requirements are given in a probabilistic form, with the maximum acceptable risk varying in 10-5-10-8 range. Confirmation of such low probabilities is possible at the statistical simulation phase only. The only method specified in West European Joined Airworthiness Requirements (JARs) is the nonparametric "pass-no-pass" approach, based on using of minimum information.

*Table 1. The requirements for maximum acceptable performance of some long-range aircraft utomatic landing systems, whether the performance under study is in or out of tolerance*

| Performance | Touchdown distance | Lateral deviation from runway centerline | Vertical speed (mod.) | Pitch angle | Bank angle | Gliding angle |
|---|---|---|---|---|---|---|
| 1-$R_{dr*}$ Exceedance probabilities (average) | $10^{-6}$ | $10^{-6}$ | $10^{-6}$ | $10^{-6}$ | $10^{-8}$ | $10^{-6}$ |
| 1-$R_{dr*}$ Exceedance probabilities (limited) | $10^{-5}$ | $10^{-5}$ | $10^{-5}$ | $10^{-5}$ | $10^{-7}$ | $10^{-5}$ |

*dr – desired

The method is based on the binomial probabilities distribution:

$$P(d/n,R)=\frac{n!}{d!(n-d)!}R^{n-d}(1-R)^{d}$$

where 1 – $R$ - estimated risk; $d$ - number of failures (out of tolerance); $n$ – sample volume; and $R$ – parameter confidence limits, defined according to Klopper-Pearson's equations:

$$\begin{cases}\sum_{r=0}^{d}\frac{n!}{r!(n-r)!}R_{\mathbf{L}}^{n-r}\left(1-R_{\mathbf{L}}\right)^{r}=1-\gamma_2\\ \sum_{r=0}^{d}\frac{n!}{r!(n-r)!}R_{\mathbf{U}}^{n-r}\left(1-R_{\mathbf{U}}\right)^{r}=1-\gamma_2\end{cases}$$

$\gamma_1+\gamma_2-1=\gamma$ - confidence belief,

where $R_L$ – lower confidence contour, $R_U$ – upper confidence contour.

The decision rule is $R_{\mathbf{L}}\geq R_{\mathrm{dr}}$ , where $R_{dr}$ is the desired value, defines the system acceptance and the rule $R_{\mathrm{U}}\leq R_{\mathrm{dr}}$ - it's rejection. From the confidence limits definition and expressions analysis it comes out that the events $r\leq d$ and $r>d-1$ with the true value $R_{true}=R_{\mathrm{dr}}$ are improbable, i.e. a system can be accepted if $R_{true}=R_{\mathrm{dr}}$, and it can be rejected if $R_{true}=R_{\mathrm{dr}}$.

An additional point is that this method, using minimum possible information, features the most broad confidence interval, therefore it requires an extremely large sample volume for the $R$ parameter assessment with the adequate statistical accuracy, or acceptance of verifiable hypothesis for this parameter value with sufficient certainty.

Thus with $R_{\mathrm{dr}}=0.999999$ (risk $1-R=10^{-6}$), the sample volume is $n$ = 2302585. The necessary amount of statistical simulation is impossible and the simulation costs are unreasonable, taking into

account that in the process of automatic landing control algorithms execution, over 10 iterations are performed.

As an alternative to a nonparametric approach, a parametric approach is suggested, based on the empirical functions probabilities distribution analysis of the performance under study (touchdown distance in the process of landing, vertical speed, bank and pitch angles, lateral deviation from the runway center line) and also on these laws approximation with some analytical expressions. A number of authors works show that in view of automatic landing systems nonlinearity, the use of certain approximation methods (Pearson, Johnson, Series families distributions) does not allow to build up an approximation with adequate accuracy [4, 5]. Therefore the new approach, taking into account the possible "discord" of these performance probabilities distribution laws is suggested. Three versions of combined distribution laws are considered: the Pareto distribution for description of the distribution laws "tail" parts, the distributions mixture and spline functions.

**Use of the Pareto distribution.** The logic justification of this approach is as follows: the empirical distribution functions provide source data. The normally distributed function in the proper scale* represents straight line (Figure 1-4). Therefore the deviations from normality in distributions "tails" (in contrast to the probability density histograms) can be seen on the distribution function graphs of the automatic landing performance under study.

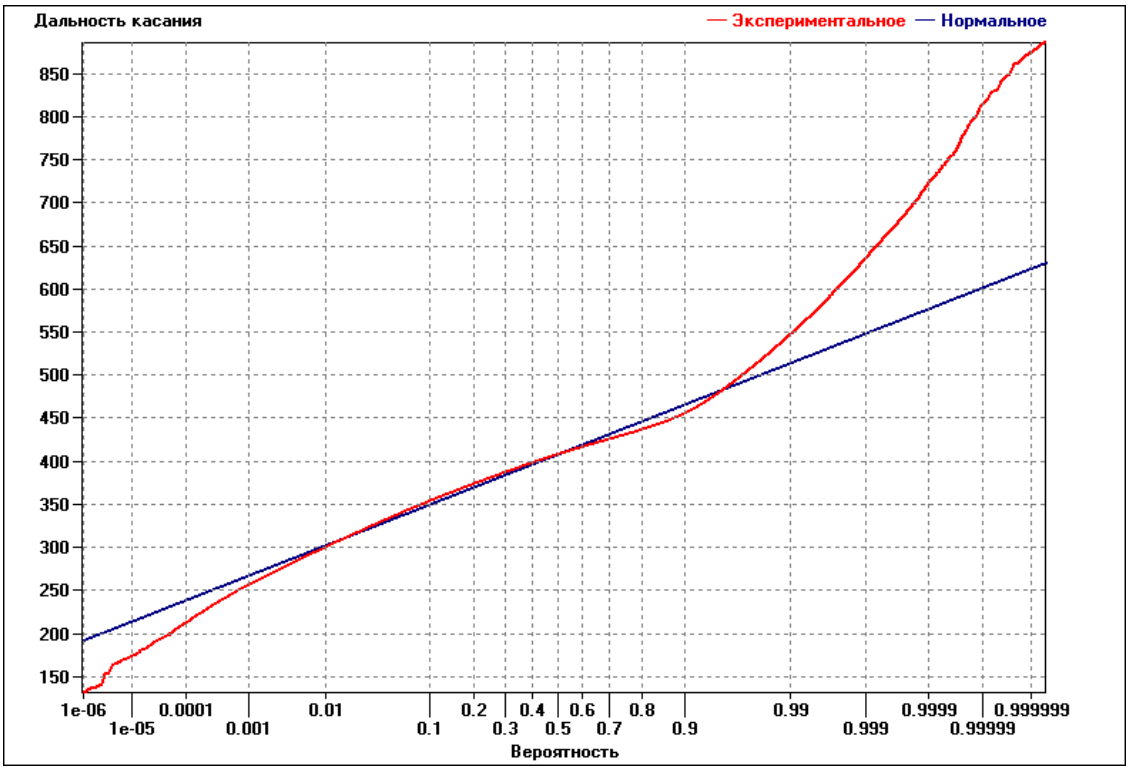


*Figure 1. Touchdown distance ($D_{td}$); Distribution function ($N=2.3\cdot10^6$)*

* It is desirable to carry out the nonlinear extension along the probability axis, i.e. to display probability interval (0.1) in the (-∞, +∞) interval for more detailed schematic analysis of what is known as "tails". This type of extension operation can be performed in many ways. If the standard normal distribution inverse function is selected for the extension operation, then, in case the empirical distribution corresponds to normal distribution, the empirical distribution diagram will be rectilinear.

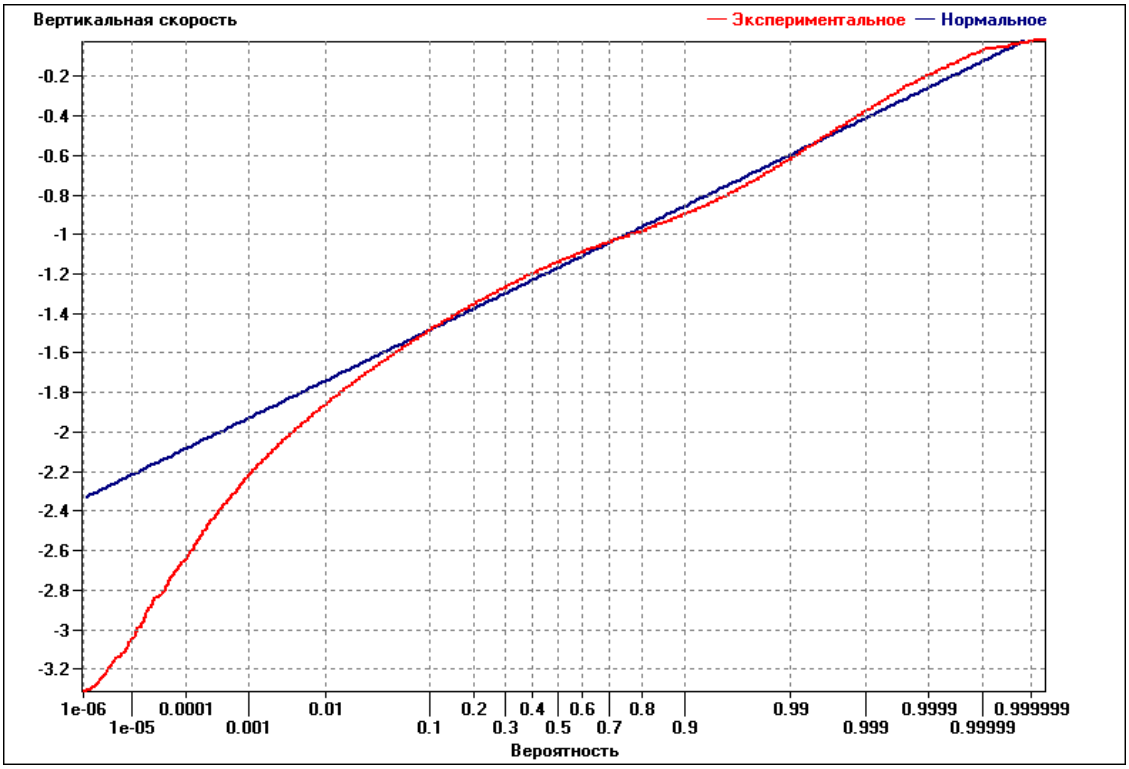


*Figure 2. Touchdown vertical speed (Vy); Distribution function ($N=2.3\cdot10^6$)*

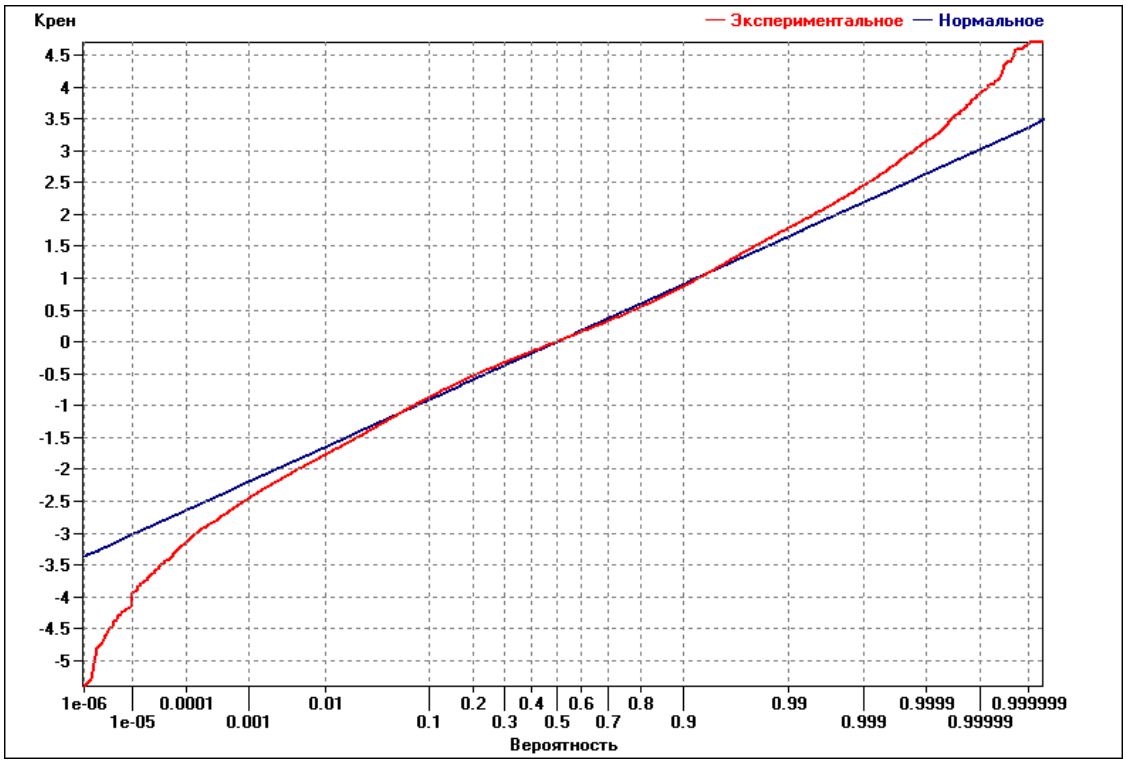


*Figure 3. Touchdown bank angle (γ); Distribution function ($N=2.3\cdot10^6$)*

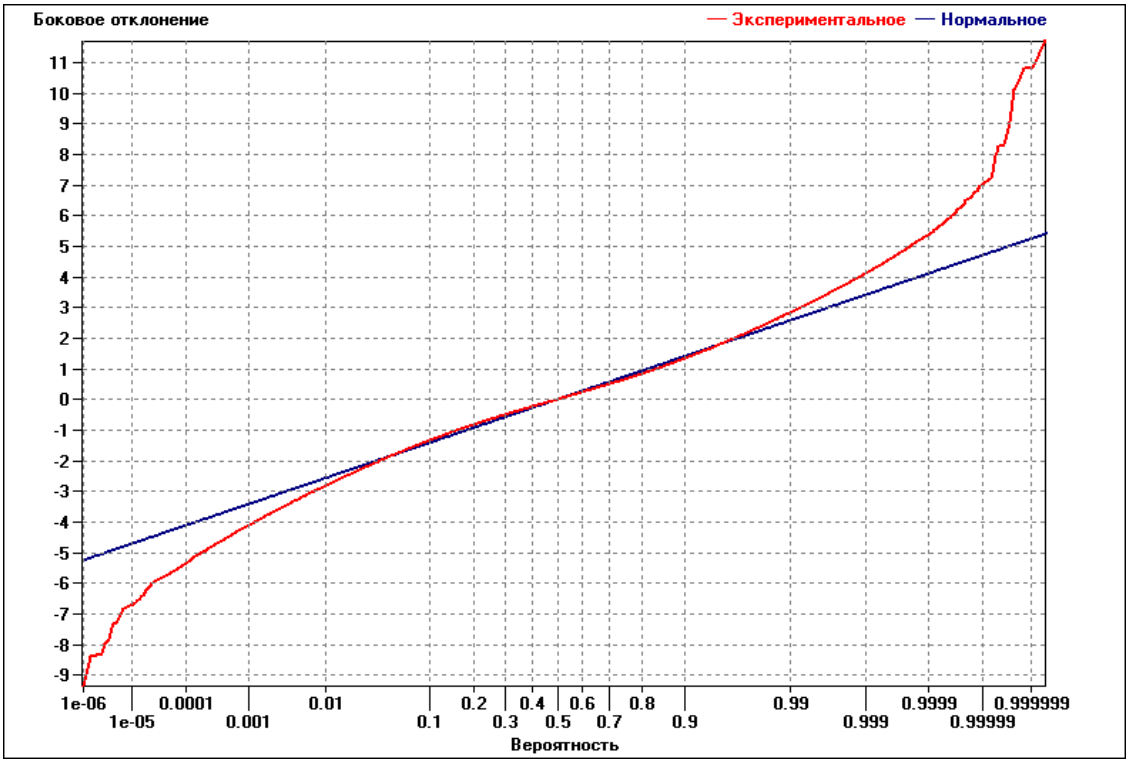


*Figure 4. Touchdown lateral deviation (Z); Distribution function (N=2.3·10$^6$)*

The distribution functions analysis was also applied in the process of Boeing 757/767 automatic landing simulation.

It is known from the theory of interpolation that the less is the interpolation interval the better is the fit to the test points. As we are not interested in the whole distribution function description, but only in the initial or final part of it, to solve the problem of the extrapolation to unobservable distributions "tails", the ordered sample volume selection, necessary for approximation, should be done with regard to the compromise between the accuracy of the selected population statistical estimations and the interpolation accuracy.

Pareto Distribution approximates a random variable distribution, larger (smaller) then some fixed value $C_0$.

The Pareto distribution is as follows:

$$F_{\text{Pareto}}(x) = 1 - \left(\frac{C_0}{x}\right)^{\alpha} \text{ for } x \geq C_0$$

The single $\alpha$ parameter moment method estimation is $\hat{\alpha} = 1 + \sqrt{1 + \left(\frac{1}{\bar{V}}\right)^2}$ where the estimated coefficient of variation (CV) is $\bar{V} = \frac{S}{\bar{x}}$; $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ ; $S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

The degree of truncation is equal to $F_c = \frac{n_{\text{sample}} - n_c}{n_{\text{sample}} + 1}$, where c – conditional, $n_c$ – number of measurements in the explored "tail" part.

Matching of the Pareto distribution with the truncated initial distribution is performed from the formula $F = F_{\text{Pareto}} \cdot (1 - F_{\text{c}}) + F_{\text{c}}$.

A prediction for unobservable "tails" of the explored parameters probability distributions can be performed on the basis of the Pareto distribution. Thus, for example, for F = 0.999999 we will determine $F_{\text{Pareto}}$ as:

$$F_{\text{Pareto}} = \frac{0,999999 - F_{\text{c}}}{1 - F_{\text{c}}}$$

By substituting the resulting value into the Pareto distribution function formula, we will find the predicted value:

$$\ln x_{\text{pr}} = \frac{\alpha \ln C_0 - \ln(1 - F_{\text{Pareto}})}{\alpha},$$

where pr – predicted.

**Use of the mixture distribution.** Mixtures of distributions were used in Japan, Western Europe and the USA in solving problems of the airplanes vertical separation safety conformance verification. The two normal mixture distributions parameters selection for the Il-96 aircraft Cat. IIIA landing distance verification problem are resulted in [5]. Thus the mixture parameters were selected by the combinatorial method on account of the empirical and theoretical moments equality condition.

The report offers a more general mixture parameters selection method based on the maximum empirical and theoretical probabilities distribution laws coincidence.

The normal distribution laws mixture $F_A(x) = \sum_{k=1}^{K} qN\left((x - m_i)/\sigma_i\right)$ with normal law parameters $\{m_i, \sigma_i, q_i\}$ - mathematical expectations, root-mean-square deviations and weighting factors (prior probabilities), when $\sum_{k=1}^{K} q_i = 1$, where $A$ is approximation and $K$ is mixture components number selected experimentally, that is used as the basis for the parametric approximation.

The mixture distribution-based parametric approximation is selected by the least-squares functional minimization of the empirical distribution function and the parametric approximation distribution function differences:

$$f_{\Sigma}\left(\{m_k, \sigma_k, q_k\}\right) = \sum_{i-1}^{n} \left\{F_{exp}(x_i) - F_A(x_i)\right\}^2 =$$

$$= \sum_{i-1}^{n} \left\{F_{exp}(x_i) - \sum_{k-1}^{K} q_k N\left((x_i - m_k)/\sigma_k\right)\right\}^2$$

under condition that $\sum_{k=1}^{K} q_i = 1$, where exp – experiment.

The distribution function behavior at the right "tail", for example, is of most interest for Touchdown Distance. It has been proven experimentally that using of a two-component mixture of normal distributions:

$$F_{exp}(x) = q_1 N\left((x - m_1)/\sigma_1\right) + q_2 N\left((x - m_2)/q_2\right),$$
$$0 \le q_1 \le 1,\ 0 \le q_2 \le 1,\ q_1 + q_2 = 1$$

is sufficient for the right "tail" approximation.

Thus, it is necessary to search the minimum with respect to the following 5 parameters: $m_1, \sigma_1, q_1, m_2, \sigma_2, q_2 = 1 - q_1$.

When plotting a graph, it is not the empirical distribution function that is plotted, but its transformation with the inverse normal distribution function.

Therefore, it is suggested to change the functional, adding a transformation by the normal distribution inverse function $N^{(-1)}\left(F_A(x)\right)$. The transformation, added to the functional, increases the summand influence on the “tails” because of extension of "tails", i.e. the function takes the form:

$$f_\Sigma\left(\{m_k, \sigma_k, q_k\}\right) =$$
$$= \sum_{i=1}^{n} \left\{ N^{(-1)}\left(F_{exp}(x_i)\right) - N^{(-1)}\left(F_A(x_i)\right) \right\}^2$$

As the points $\{x_i = 1, \ldots, n\}$ lie in the central range more densely, the approximating curve will be more exact in the central range, but less exact at the "tails". Hence, this functional considers the empirical probability density indirectly, i.e. the approximation is performed "statistically". The functional $f_\Sigma$ equivalent type in the integrated form - Stieltjes integral is as follows:

$$f_\Sigma\left(m_1, \sigma_1, q_1, m_2, \sigma_2, q_2\right) =$$
$$= \int \left\{ N^{(-1)}\left(F_{exp}(x)\right) - N^{(-1)}\left(F_A(x)\right) \right\}^2 dF_{exp}$$

Besides the functional in the integrated form can be written as a conventional "functional":

$$f_\int\left(m_1, \sigma_1, q_1, m_2, \sigma_2, q_2\right) =$$
$$= \int_{x_1}^{x_n} \left\{ N^{(-1)}\left(F_{exp}(x)\right) - N^{(-1)}\left(F_A(x)\right) \right\}^2 dx$$

It will lead to terms influence minimization in the places, where the density of points is high. During trapezoidal integration, for example, in the places where the points $\{x_i i=1,\ldots,n\}$ lie more densely, $\Delta x_i$ will be small, and, thereby, the degree of the integration element influence will decrease in this range.

Table 2 demonstrates the agreement between An-148 Touchdown Distance theoretical and experimental distribution function basic statistical performance. The agreement between the moments serves as assurance of the method suggested.

*Table 2. Agreement between An-148 Touchdown Distance theoretical and experimental distribution function basic statistical performance*

| Number of tests | Distribution | Moments | | | |
|---|---|---|---|---|---|
| | | Mathematical expectation | Root-mean-square | Asymmetry exponent | Excess exponent |
| 300 000 | Experiment | 400.713 | 40.374 | ≈ 0 | 4.711 |
| | Mixture | 400.423 | 40.383 | ≈ 0 | 4.729 |
| 600 000 | Experiment | 400.720 | 40.317 | ≈ 0 | 4.686 |
| | Mixture | 400.509 | 40.263 | ≈ 0 | 4.775 |
| 1 000 000 | Experiment | 400.689 | 40.339 | ≈ 0 | 4.654 |
| | Mixture | 400.427 | 40.302 | ≈ 0 | 4.703 |

**Use of the spline-approximation.** We will apply the spline-approximation to the function $S_n^{exp}(x) = N^{-1}\left(F_n^{exp}(x)\right)$.

Let us assume, that $S_n^{exp}(x_i)$ experimental function values are determined with an error in the form of zero mathematical expectation variable with the dispersion $D_i^S = D\left(S_n^{exp}(x_i)\right)$. Then the approximating function deviation estimation $S_n^A(x_i)$ from experimental function $S_n^{exp}(x_i)$ can be represented as

$$\sum_{i=1}^{n} \frac{1}{D_i^S}\left(S_n^{exp}(x_1) - S_n^A(x_i)\right)^2 = c \cdot n \qquad (1)$$

where the $c$ constant is selected experimentally.

A variety of approximating functions can meet this condition. The spline-function theory supposes that a function with the smoothness property should be selected from this variety of functions. In the spline-function theory, the functional of the function smoothness $S(x)$ is given by the integrated square $S''(x)$

$$I_2(S) = \int_{x=x_1}^{x_n} \left(S''(x)\right)^2 dx \qquad (2)$$

Thus, the approximating function $S_n^A(x_i)$ should minimize functional (2) under condition of (1).

When introducing the Lagrangian multiplicity $\lambda$, we will reduce the conditional minimization problem to the unconditional problem by two variables $S_n^A$ and $\lambda$:

$$I_2\left(S_n^A\right) + \lambda \cdot \left[\sum_{i=1}^{n} \frac{1}{D_i^S}\left(S_n^{exp}(x_i) - S_n^A(x_i)\right)^2 - c \cdot n\right]$$

For values of the experimental function $S_n^{exp}(x_i)$ it is possible to indicate approximately $D_i^S$ dispersions.

It is known that the experimental function spreads distribution follows the binomial law:

$F_n^{exp}(x)$ with $D_i^F = F_n^{exp}(x_1)\left(1 - F_n^{exp}(x_i)\right)/n$ dispersion.

When defining the $D_i^S$ dispersion of the $S_n^{exp}(x_i)$ function it is necessary to consider $S_n^{exp}(x) = N^{-1}\left(F_n^{exp}(x)\right)$ inverse transformation.

Let's consider the transformation linear approximation:

$$S_n^{exp}(x_i) = N^{-1}\left(F_n^{exp}(x_i)\right) = N^{-1}\left(F^{true}(x_i) + \sigma_i^P\right) = \\ = N^{-1}\left(F^{true}(x_i)\right) + \frac{\sigma_i^P}{dN/dt} 0 \cdot \left(\sigma_i^P\right), \qquad (3)$$

where the normal distribution derivative is taken at $t = N^{-1}\left(F^{true}(x_i)\right)$ point, and $F^{true}$ - the truth distribution function.

Hence, approximately $D_i^S = \frac{D_i^P}{\left(dN/dt\right)^2}$, where $dN/dt = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$. The complete expression of random value $S_n^{exp}(x_i)$ dispersion has the following form:

$$D_i^S = \frac{F^{true}(x_i) \cdot \left(1\text{-}F^{true}(x_i)\right)}{n\left(\frac{1}{\sqrt{2\pi}} e^{-t^2/2}\right)} = \\ = 2\pi e^{t^2} \cdot F^{true}(x_i) \cdot \left(1\text{-}F^{true}(x_i)\right)/n \qquad (4)$$

As the truth function distribution $F^{true}(x_i)$ value is unknown, we take instead as a first approximation the experimental distribution function $F_n^{exp}(x_i)$ value:

$$D_i^S = 2\pi e^{t^2} \cdot F_n^{exp}(x_i) \cdot \left(1\text{-}F_n^{exp}(x_i)\right)/n \qquad (5)$$

Let's reduce the functional (3) with the fixed parameter to the following form:

$$I_2\left(S_n^A\right) + \sum_{i=1}^{n} \frac{1}{D_i^S/\lambda}\left(S_n^{exp}(x_i) - S_n^A(x_i)\right)^2 \qquad (6)$$

i.e. $p_i = D_i^S/\lambda$

With the fixed $\lambda$ parameter the summand $\lambda \cdot c \cdot n$ is also fixed, therefore it can be ignored during minimization.

As initial information for construction of the smoothing spline-function it is necessary:

- to set the values of arguments, in which the experimental distribution function is known,
- to set the experimental distribution function values,
- to define the weights,
- to set the boundary conditions.

At the extremities, the distribution function $F_n^{exp}(x)$ asymptotically approaches to *0* at the left extremity and to *1* at the right extremity. The distribution function transformation $S_n^{exp}(x) = N^{-1}\left(F_n^{exp}(x)\right)$ tends to extension at the "tails" i.e. the "tails" flexon approaches to *0*.

Therefore, the zero values of the flexons $d^2 S_n^A(x)/(dx)^2$ with $x = x_1$ and $x = x_n$ are set as the boundary conditions. The arguments grid is not necessarily proportional, and it can be variable. When working with the splines the mandatory requirement is that the arguments should not have repeats, i.e. zero step of the grid along the argument axis is not acceptable.

In this paper we use the approximating function representation through the parameters of: $M_I = d^2 S_n^A(x_i)/(dx)^2$ form:

$$S_n^A(x) = S_n^A(x_i)\cdot(1\text{-}t) + S_n^A(x_{i+1})\cdot$$
$$\cdot\, t - \frac{h_i^2}{6} t(1-t)\cdot\left[(2-t)M_i + (1+t)M_{i+1}\right], \qquad (7)$$

$$x \in [x_i, x_{i+1}],\ i = 0,1,\ldots,n\text{-}1,$$
$$h_i = x_{i+1} - x_i,\ M_i = S_n^{A}{}''(x_i),$$
$$t = \frac{x - x_i}{h_i},\ 0 \le t \le 1.$$

The required parameters are $S_n^A(x_i)$ and $M_i$. The parameters are defined from: the (6) functional minimization condition, $S_n^A{}'(x_i - 0) = S_n^A{}'(x_i + 0)$ spline-function first-order derivate continuity requirement and $S_n^A{}''(x_0) = S_n^A{}''(x_n) = 0$ boundary conditions. The approximation is performed for the empirical distribution function right "tail", beginning from the point, where the experimental probability itself has a reasonably good accuracy $F_n^{exp}(x) \ge 0.999$ .

Using the above-described procedure, the following estimations were conducted:

- determination of the probabilities spreads dispersions,
- transformation of the empirical distribution function with the aid of the function, that is opposite to the normal distribution function,
- re-estimation of the spreads dispersion (4),
- definition of the spline-function from (1) and (2) problems solution with the parameter $c$ = 0,1.

Table 3 demonstrates the basic statistical performance agreement between An-148 Touchdown Distance theoretical and experimental distribution functions. The moments agreement serves as the suggested method assurance.

As the An-148 automatic landing statistical simulation is performed in "portions", the capability occurs to stop the simulation process, if required, and hence, to reduce its time and costs.

Table 4 demonstrates the results of the Touchdown Distance prediction, performed using all three methods per 1 million of realizations.

*Table 3. Basic statistical performance agreement between An-148 Touchdown Distance theoretical and experimental distribution functions*

| Number of tests | Distribution | Moments | | | |
|---|---|---|---|---|---|
| | | Mathematical expectation | Root-mean-square | Asymmetry exponent | Kurtosis |
| 300 000 | Experiment | 400.713 | 40.374 | ≈ 0 | 4.711 |
| | Splines | 400.705 | 40.368 | ≈ 0 | 4.686 |
| 600 000 | Experiment | 400.720 | 40.317 | ≈ 0 | 4.686 |
| | Splines | 400.11 | 40.312 | ≈ 0 | 4.669 |
| 1 000 000 | Experiment | 400.689 | 40.339 | ≈ 0 | 4.654 |
| | Splines | 400.605 | 40.376 | ≈ 0 | 4.631 |

*Table 4. Results of the Touchdown Distance prediction, performed using all three methods per 1 million of realizations*

| Number of realizations | Approximation method | Prediction result (m) | |
|---|---|---|---|
| 300 000 | Pareto | 839.673 | The experimental value per 1 million of realizations - 814,9 m |
| | Mixtures | 857.746 | |
| | Splines | 826.03 | |
| 600 000 | Pareto | 830.435 | |
| | Mixtures | 848.35 | |
| | Splines | 825.08 | |
| 1 000 000 | Pareto | 815.865 | |
| | Mixtures | 829.842 | |
| | Splines | 818.67 | |

Thus, the best approximation accuracy is provided by the Pareto distribution and the splines. In addition, the deviation between the results for these two methods is of small importance (max 13m), and it decreases with the realization volume growth. It also should be noted, that all considered methods give some experimental value "overestimation", providing the achieved results assurance.

On the basis of approximation accuracy analysis, the two versions were chosen: the spline-functions and the Pareto distribution, leading to almost similar results.

Spline-functions describe the random value variation distributions across the full-range, while the Pareto distribution describes distributions only at the bounded "tail" parts for the random values, which are larger (or lesser) then some certain values. Therefore the first form of the approximation has more credibility then the second. However, the use of the Pareto distribution enables to estimate the approximation accuracy analytically. Both versions were used to solve the problem of extrapolation on the distributions unobservable "tails", providing the rapid analysis procedure feasibility, which enables, without performing any comprehensive mathematical simulation, to

make a decision whether it is necessary to adjust the control law, or to increase the volume of the simulation further (Figure 5).

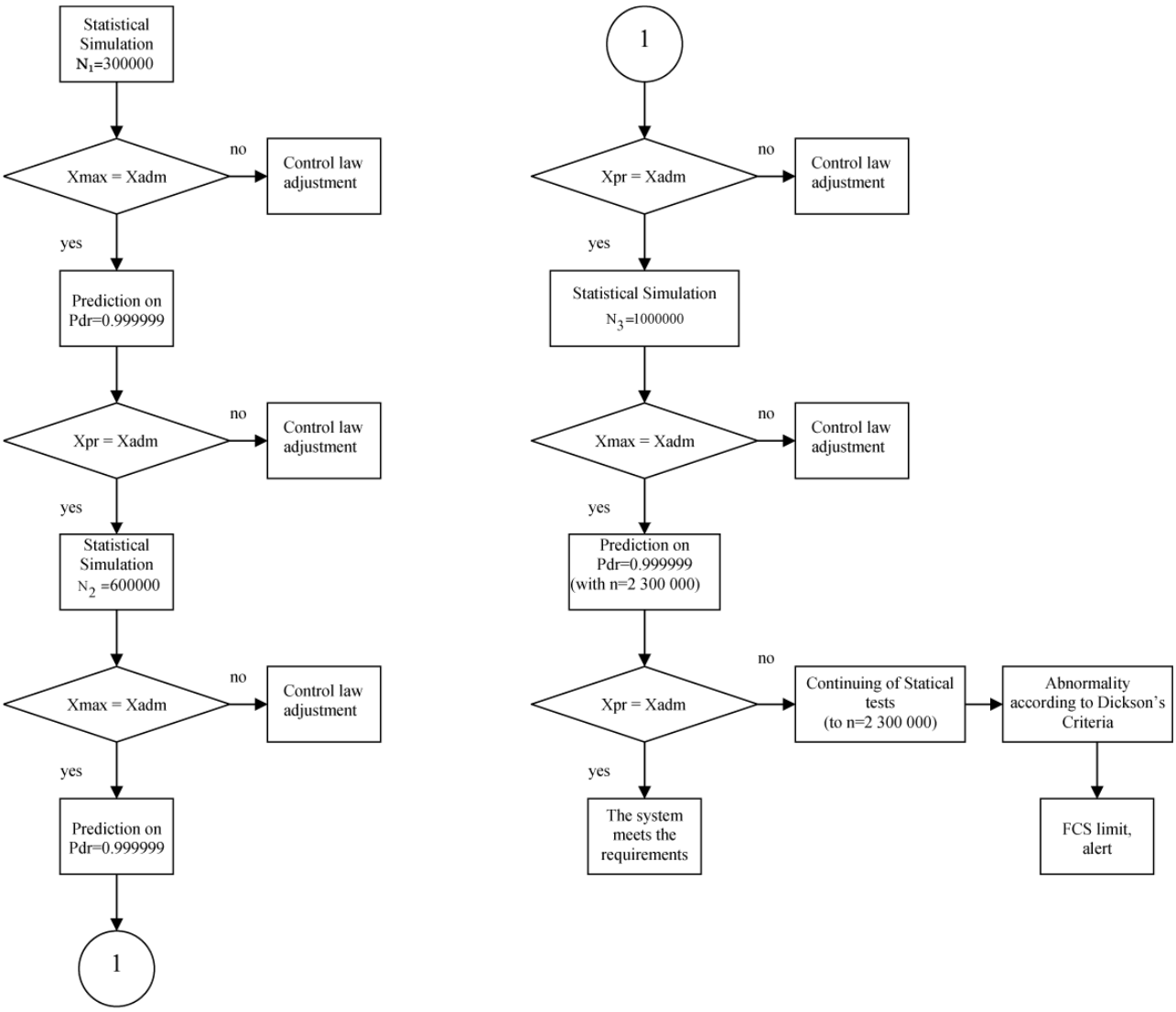


*Figure 5. Rapid analysis flow chart*

In the process of final selection of the control law, the single use of the «pass-no-pass» method is possible.

The approach suggested was widely approved during the An-148 automatic landing system simulation. The results of the airplane flight tests have confirmed the received results. The airplane equipped with the САУ-148 (FCS-148) was certified for ICAO Cat. IIIa landing.

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## REFERENCES


[1] Technical Specifications for the Transport Category Airplanes, performing All-Weather Operations. ARMAK, 2003.

[2] Certification Specifications for All Weather Operations (CS AWO). European Aviation Safety Agency, 2002.

[3] Criteria for Approval of Category III Landing Weather Minima FAA AC-120-28-D, 1999.

[4] S. P. Kryukov, S. D. Bodrunov, L. N. Aleksandrovskaya, A. G. Kuznetsov etc., Methods of hazards analysis and estimation in the problems of complex technical systems safety management, Aerospace Equipment Corporation, St. Petersburg, 2007.

[5] L. N. Aleksandrovskaya, A. E. Ardalionova, V. G. Borisov, V. N. Masur, S. V. Hlgatyan, The moment method in a problem of aircraft safety requirements means of compliance at automatic landing// MIEA Papers. Navigation and control of flight vehicles, issue 6, 2013.

[6] A. Shakarian, Application of Monte-Carlo techniques to the 757/767 auto land dispersion analysis by Simulation. AIAA Guidance and Control Conference, New York, USA, 1983.

[7] Vertical Separation at High Altitudes RGCSP-WG/WHL/8 - WP/7, 8.

[8] Review of the General Concept of Separation Panel Sixth Meeting.

[9] D. Himmelblau, Application nonlinear programming, MIR publishing house, Moscow, p.168, 1975.

[10] G. I. Ivchenko, Yu. I. Medvedev, Introduction to mathematical statistics, LKI publishing house, Moscow, 2010.

[11] N. S. Bakhvalov, N. P. Zhidkov, G. M. Kobelkov, Numerical methods. Nauka publishing house, Moscow, 1987.

[12] Y. S. Zavyalov, B. I. Kvass, V. L. Miroshnichenko, Spline functions methods. Nauka publishing house, Moscow, Fizmatlit,1980.